

AN INTRODUCTION TO BRAILLE MATHEMATICS

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AN INTRODUCTION TO BRAILLE MATHEMATICS

by

HELEN ROBERTS
BERNARD M. KREBS
BARBARA TAFFET

Based on
the Nemeth Braille Code for Mathematics
and Science Notation, 1972

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FOREWORD

An Introduction to Braille Mathematics has been designed as an easily assimilated presentation of the special symbols and complex rules and procedures laid down in the **Nemeth Code for Mathematics and Science Notation**. Lessons and exercises have been arranged in a smooth, step-by-step progression; wherever possible, rules and guidelines have been couched in familiar terms rather than the highly technical vocabulary of the mathematics expert. This style of presentation should enable the braille transcriber with a general knowledge of mathematical notation to become fully conversant with the provisions of the code and to make a meaningful contribution in this vital area of specialized service.

The opportunity for blind persons to achieve their educational and professional goals depends to a great extent on the availability of basic texts and reference material in a suitable medium. A well-trained braille transcriber is the prime resource in the production of textbook material. The National Library Service for the Blind and Physically Handicapped, formerly the Division for the Blind and Physically Handicapped, Library of Congress, has lent its support and encouragement in the development and publication of this comprehensive manual as an expression of its gratitude to the volunteers for their indispensable services in helping blind individuals reach their highest potential.

Work on this manual was begun under the inspired leadership of Helen Roberts, whose thorough expertise was gained through firsthand experience in the development of the Nemeth code. Helen's warmth and dedication as a volunteer transcriber and exceptional teacher touched everyone she met. Her untimely death halfway through this project was, and still is, felt as a cruel loss by all who knew her. We dedicate this book to her memory, which will live on in the work of those who serve and in the lives of those who are being served by her endeavors.

By good fortune, Barbara Taffet, a friend, associate, and student of Helen's, stepped forward to fill the void. Her understanding of the Nemeth code and her painstaking efforts have carried this work to completion; her service to transcribers and blind people is worthy of the highest praise.

For their invaluable assistance, we gratefully acknowledge:

The late Robert S. Bray, former Chief of the Division for the Blind and Physically Handicapped, and Frank Kurt Cylke, Director of the National Library Service for the Blind and Physically Handicapped, for their devotion to expanding the horizons of blind people through the written word and for their active support of this publication;

Donna Pastore, for contributing her knowledge and expertise while working closely with the authors;

Maxine B. Dorf and Alice Mann, for their suggestions and evaluation on the work in progress;

Virginia Brooks Scharoff, for her generous and far-reaching support, cooperation, and encouragement;

Kay Hollander, Deborah Gordon, Nancy Schattner, and Beatrice Scheps, for their competent and rigorous service in preparing working drafts of the manuscript for publication;

The Pinellas Braille Group of St. Petersburg, Fla., Volunteer Braille Services of Washington, D.C., the Industrial Home for the Blind, and the Jewish Guild for the Blind, for lending their staff, equipment, and facilities to aid in preparation of this manuscript.

**Bernard M. Krebs, Librarian
Jewish Guild for the Blind**

CERTIFICATION

The Course

The homework exercises in this manual are designed to prepare certified literary braillists for the Library of Congress examination leading to certification in braille mathematics transcribing. Students may study independently, train under the supervision of a qualified braille mathematics instructor, or enroll in the Library of Congress free correspondence course.

Students enrolled in this correspondence course complete the homework exercises and return them for review and evaluation by an instructor at the Library of Congress. After the exercises have been reviewed, the instructor returns a detailed report pointing out errors, making helpful comments, and giving a new assignment.

The Application

Candidates may use the application on page vii of this manual to enroll in the free correspondence course or to apply directly for the examination and certification in braille mathematics. Students wishing to enroll should complete the homework exercises for Lesson 1 and return them with the application form to the Library of Congress.

Mailing

Please pack exercises and test materials between sheets of cardboard and return them in a large envelope. Do not fold pages, and be sure that test pages are packed in proper order and securely tied. Send all materials to Volunteer Training Section, National Library Service for the Blind and Physically Handicapped, Library of Congress, Washington, D.C. 20542. The instructor will return reports and materials submitted with an addressed postage-free mailing label.

APPLICATION

Date_____

I hereby request that (check appropriate box) :

☐ I be enrolled in the Library of Congress correspondence course in the transcription of braille mathematics.

☐ My test in the transcription of braille mathematics be examined for certification.

Name in full_____
(Indicate whether Mr., Miss, or Mrs. If married, give husband's initials)

Address_____

ZIP

Telephone number (give area code)_____

Name as it should appear on certificate (please print)

Date of certification in literary braille at the Library of Congress_____

Approximate number of braille pages you have transcribed:

Literary braille_____

Mathematics braille_____

Are you familiar with textbook format? Yes_____ No_____

Group affiliated with_____
(If enrolling in correspondence course, put Library of Congress)

Address of group_____

Name of group chairman_____

Signed_____

LESSON 1

ORIENTATION

§1. Philosophy: The braille code for mathematics is especially designed for the representation and transcription of mathematical and scientific notation. Its purpose is to convey, as accurately as possible, a clear conception of the printed text to the braille reader. Using braille indicators in conjunction with the 63 braille characters, this code is capable of providing equivalent symbols for the hundreds of mathematical and scientific ink-print signs now in use and yet to be devised. The one-to-one correspondence between braille and ink-print symbols makes it possible to produce an accurate transference from ink-print to braille or from braille to ink-print.

§2. Nontechnical and Technical Texts:

a. Nontechnical Texts: For the purposes of the Nemeth code, a nontechnical text is any work in which no mathematical or scientific notation appears. Such texts must be transcribed in accordance with the rules of English braille.

b. Partially Technical Texts: A partially technical text is a science book or a work which utilizes a small number of mathematical symbols. Such texts must be transcribed according to the rules of English braille. However, when the replacement of symbols by words is not practical or possible, or when the mathematical display is used for solving equations or performing computations, the symbols and rules of the Nemeth code must be used, and the braille reader must be so advised. A list of the special symbols being used must be placed at the beginning of each braille volume in which they occur.

c. Technical Texts: A technical text is a work in the field of mathematics, statistics, physics, or chemistry. Such texts must be transcribed entirely according to the rules of the Nemeth code. The symbols and rules of the Nemeth code must also be used in works in other fields which make substantial use of mathematical or scientific notation. Since several revisions of the Nemeth code are in use, a transcriber's note stating that the text is transcribed in Nemeth code and giving the year the code was adopted must be included at the beginning of each volume.

NUMERALS AND THE NUMERIC INDICATOR

§3. Representation of Arabic Numerals: In the transcription of a technical text, digits are represented in two ways:

a. English Braille Numerals: English braille numerals represented by the letters "a" through "j" must be used for all Arabic numerals appearing on title pages, except those used in conjunction with mathematical symbols. English braille numerals must also be used for page numbers at the corners of pages and at the ends of page-separation lines.

b. Nemeth Code Numerals: In the Nemeth code, the 10 Arabic digits are represented by the letters "a" through "j" dropped to the lower part of the braille cell. Except as stated in a above, Nemeth code numerals must be used to represent all Arabic numerals in the body of the print text, forewords, contents pages, introductions, bibliographies, and indexes. They must also be used for all chapter, theorem, exercise, problem, and page-reference numbers.

1	2	3	4	5	6	7	8	9	0
⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠

§4. Numeric Indicator:

In a technical text, the numeric indicator must be used before a numeral following a space or occurring at the beginning of a braille line. However, the numeric indicator must not be used after a space intentionally inserted to divide a numeral into short regular groups of digits.

(1) 7

(2) 32

(3) 5980

(4) 9 inches and 15 inches are 2 feet.

(5) 931684572 can be divided into groups of three digits each: 931 684 572

PUNCTUATION MARKS AND THE PUNCTUATION INDICATOR

§5. **Punctuation:** Unless otherwise stated, words and English braille numerals must be punctuated according to the rules of English braille. Mathematical expressions such as Nemeth code numerals, mathematical symbols, etc. must be punctuated in accordance with the rules of Nemeth code.

§6. **Punctuation Marks and Use of the Punctuation Indicator:** Since numerals and punctuation marks are represented by identical lower-cell braille symbols, the punctuation indicator must be used before one or before a sequence of two or more of the punctuation marks listed below when they *follow* a numeral or any other mathematical expression.

Punctuation Indicator

Punctuation Marks

Apostrophe

'

Colon

:


Exclamation Point

!

Period

.

Question Mark

? 

Quotation Marks


Outer (opening and closing)

“ ” 

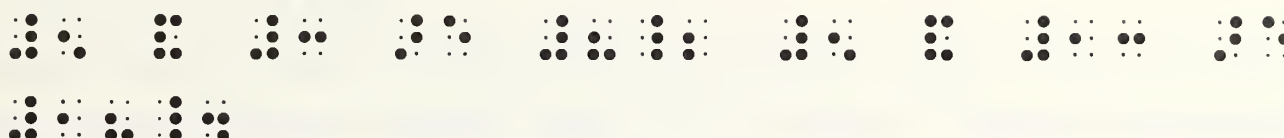
Inner (opening and closing)

‘ ’ 

Semicolon

; 

(1) 5 and 3 are 8; 5 and 13 are 18.




(2) “Do 6 and 4 equal 10?”



§7. Punctuation Marks and Nonuse of the Punctuation Indicator:

Punctuation Marks

Hyphen


- 

Dash (short)

— 


Comma

Literary


, 

Mathematical

American*

, 

Continental*

• 

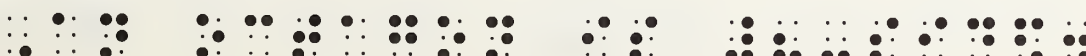
*Although the symbols for the American and Continental commas differ in print, the difference is not shown in braille. A transcriber's note must be included at the beginning of the braille text to inform the reader of the Continental usage in the ink-print edition.

a. **Hyphen and Dash:** The punctuation indicator must not be used before the hyphen or the dash. Unless otherwise stated, a word or part of a word joined to a numeral by the hyphen or the dash must be contracted and punctuated according to the rules of English braille. A space must be left between a hyphen and an adjoining dash.

(1) He bought a 6-cent stamp.



(2) An octagon is 8-sided.



- (3) 1-ary is the same as unary.

- (4) 12—one dozen—costs 48 cents.

- (5) It is 5- —as shown—not 6-sided.

The figure consists of 12 small diagrams arranged horizontally. Each diagram shows a pattern of black dots on a grid. The patterns are as follows:

- Diagram 1: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2).
- Diagram 2: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus a dot at (3,1).
- Diagram 3: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1) and (3,2).
- Diagram 4: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1), (3,2), and (3,3).
- Diagram 5: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1), (3,2), (3,3), and (3,4).
- Diagram 6: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1), (3,2), (3,3), (3,4), and (3,5).
- Diagram 7: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1), (3,2), (3,3), (3,4), (3,5), and (3,6).
- Diagram 8: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), and (3,7).
- Diagram 9: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (3,7), and (3,8).
- Diagram 10: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (3,7), (3,8), and (3,9).
- Diagram 11: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (3,7), (3,8), (3,9), and (3,10).
- Diagram 12: A 2x2 grid with dots at (1,1), (1,2), (2,1), and (2,2), plus dots at (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (3,7), (3,8), (3,9), (3,10), and (3,11).

b. **Mathematical Comma:** The mathematical comma \therefore must be used for a comma occurring in a long numeral or following a numeral or other mathematical expression. The punctuation indicator must not be used before the mathematical comma.

- (1) In 1,234,567, 1, 3, 5, and 7 are odd numbers.

- (2) Add: 10, 20, and 30.

- (3) Chapter 5, Exercise 2, Page 23.

- (4) 1.935

(the Continental comma is shown in print)

Note: In a sequence of punctuation marks, the punctuation indicator must precede the first punctuation mark following a mathematical expression. However, it is omitted before the mathematical comma, the hyphen, or the dash.

- (5) 6-, 7-, and 8-sided figures.

- (6) It is “plus 3,” not “minus 3.”

- (7) In "Figure 4", find the area.

(1) The polygon is 5-sided, not 6-sided.

The figure consists of 10 sub-diagrams arranged horizontally, each showing a 5x5 grid of dots. Black dots represent the 'on' state of a system, while white dots represent the 'off' state. The sequence shows a pattern that starts as a small cluster of 4 dots in the first diagram and grows into a complex, fractal-like structure of 24 dots in the tenth diagram. The growth follows a recursive pattern where existing clusters are replicated and combined to form new, larger clusters.

(numerals on a title page)

§8. Signs of Operation: A few signs of operation and their braille equivalents are listed below. Since the minus sign and the hyphen are represented by the same symbol in both print and braille, the student must determine the meaning of the symbols from their context.

5

Minus or Plus	\mp	
Plus Followed by Minus	$+-$	
Minus Followed by Plus	$-+$	
Minus Followed by Minus	$--$	

§9. **Spacing and Punctuation With Signs of Operation:** Unless otherwise stated, a sign of operation must be unspaced from its related mathematical terms. A sign of operation is a mathematical symbol and must be punctuated accordingly.

- (1) What does $16 + 4 + 100$ equal?

- (2) Add: $6 + 4$, $5 + 3$, and $10 + 2$.

- (3) $9 - 3 - 2$

- (4) $5 \times 6 \times 10$

- (5) $5 \cdot 6 \cdot 10$

- (6) $24 \div 3 + 2$

- (7) 38 ± 7

- (8) 38 ∓ 7

- (9) $10 + - 5$


- (10) $10 - + 5$

- (11) What is the meaning of $+$, $-$, \times ?


- (12) The symbol for "plus" is " $+$."


INTRODUCTION TO SIGNS OF COMPARISON

§10. Signs of Comparison: A few signs of comparison and their braille equivalents are listed below.


Equals \equiv 

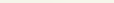
Greater Than (is greater than)


With Straight Sides $>$ 

With Curved Sides 

Less Than (is less than)

With Straight Sides < 

With Curved Sides 

Proportion (as) :: 

Ratio (is to) :

§11. Spacing and Punctuation With Signs of Comparison: A space must be left between a sign of comparison and a sign of operation or any other expression which precedes or follows it. A sign of comparison is a mathematical symbol and must be punctuated accordingly. A space must not be left between a sign of comparison and a punctuation mark which applies to it.

(1) Can $3 \pm 1 = +4$ and $+2$?

The figure shows a 4x10 grid of dots. The dots are arranged in a regular pattern, with some dots missing in the top row and bottom row to form a shape resembling a stylized 'H' or a grid with missing corners.

$$(2) \quad 19 + 6 = 25$$

(3) $7 > 4 > 3$

(4) $7 \succ 4 \succ 3$

(5) $8 + 10 < 19$

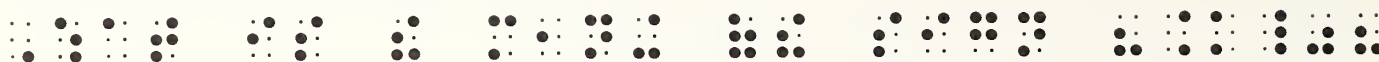
(6) $5 < 9 < 11$

(7) $2 : 4 :: 6 : 12$

(8) Use $=$, $>$, or $<$.



(9) What is the meaning of the sign “=”?



FORMAT

§12. General Rules for Format: The principles provided in the *Code of Braille Textbook Formats and Techniques* should be followed unless specific format provisions are given in the braille code for mathematics. In the transcription of technical texts, a 41-cell braille line is recommended.

§13. Margins for Itemized Material With No Subdivisions:

a. When unsubdivided itemized material, including exercises or outlines, is numbered or lettered, the number or letter must begin in cell 1, and all runovers must begin in cell 3. If the material contains more than one paragraph, each new paragraph must begin in cell 5, and any runovers must begin in cell 3.

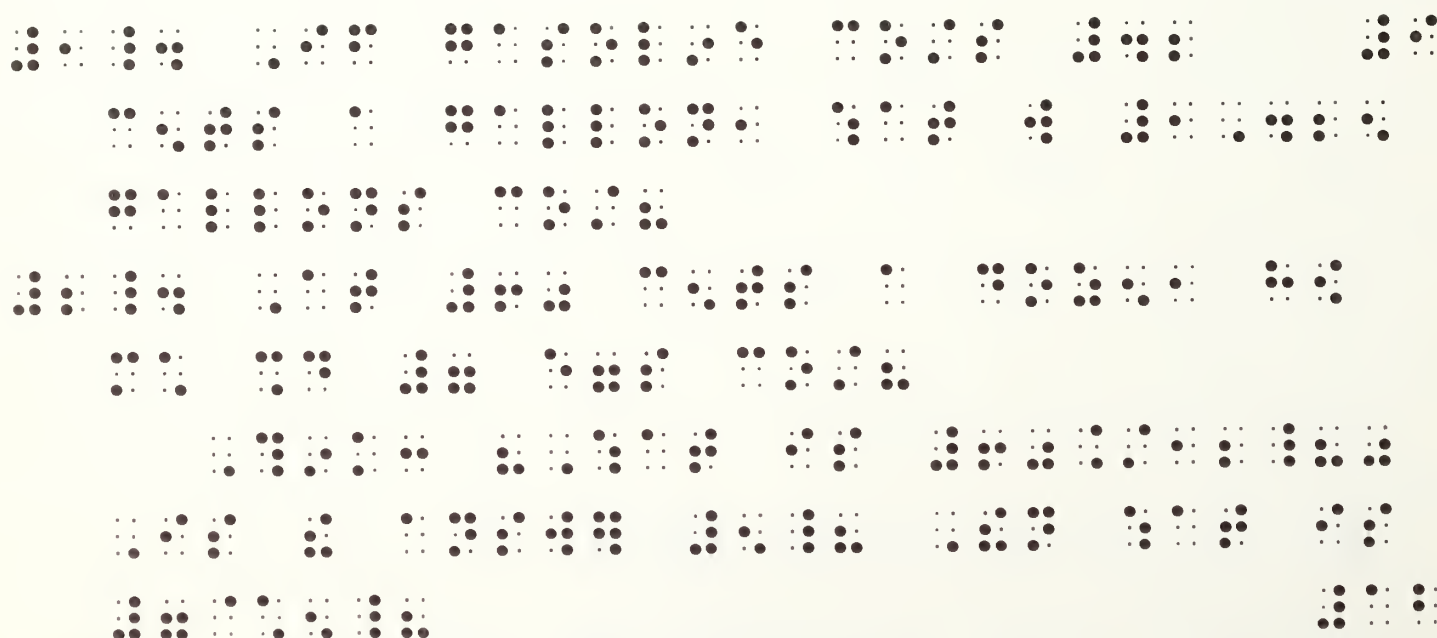
(1)

9

1. If gasoline costs 42 cents a gallon, what will 1,425 gallons cost?

2. At 60 cents a dozen, how much should 7 eggs cost?

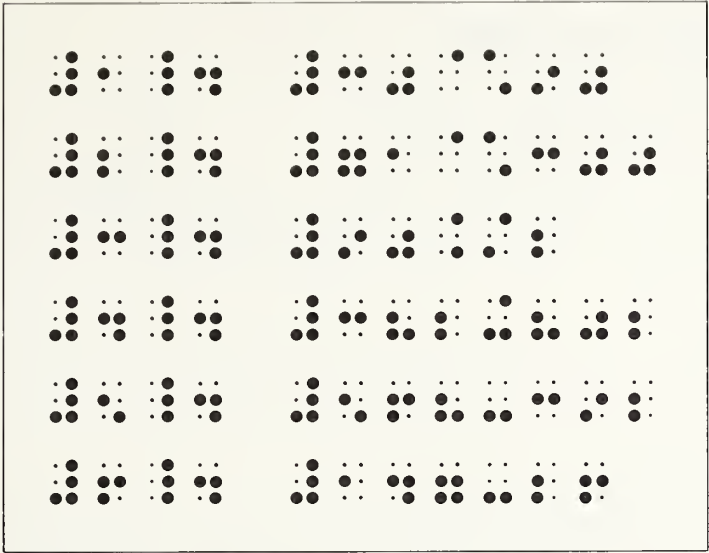
Think: “What is $60 \div 12$?” Is the answer 5? Then what is 7×5 ?



b. When unsubdivided itemized material is arranged side by side across the page in print, the braille format must be changed so that all numbers or letters start in cell 1.

(1)

1. 30 × 90	2. 71 × 300	3. 90 ÷ 2
4. 382 + 802	5. 568 — 392	6. 147 — 26



§14. Division of Mathematical Expressions Between Braille Lines:

- a. A mathematical expression, such as a long numeral or an equation, must not be divided between braille lines or braille pages. If there is insufficient space on a line to accommodate the expression, this space must be left blank, and the entire expression must be brought down to the next line.
- b. A hyphenated expression, such as “6-sided,” containing one or more mathematical components must not be divided between braille lines or braille pages.

HOMework

Prepare the homework for each lesson in the following way:

- (1) Use 11 by 11½-inch paper, and braille across the 11½-inch width of the page.
- (2) Use 41 cells and 25 lines.
- (3) Treat the word EXERCISE and the appropriate exercise number as a heading. Center it on the first line of the first page of the homework. Skip a line after the heading and start the first problem on line 3. The work on all succeeding pages should begin on line 1.
- (4) Write your name and address in print and in braille on the final page of each exercise. If space does not permit, attach a separate page with this information.
- (5) Page numbers at the upper right-hand corners and at the ends of new page separation lines should correspond to the print page numbers on which the homework exercises appear. If more than one braille page is required to complete one print page, the page number should be preceded by the letters “a,” “b,” etc. The running braille page numbers appearing in the lower right-hand corners of the braille pages should be numbered consecutively, starting with number 1 in each lesson. When there is room to start a new ink-print page in the middle of a braille page, this should be done. **Note:** The upper-cell numerals of English braille must be used for page number designations in accordance with §3a.

EXERCISE 1

1. I paid 90 cents for 12 oranges and found that 3 of them were spoiled. How much did each good orange cost?
2. He came to see us for 10 minutes, and left at 3 o'clock.
3. Round 374 and 962 to the nearest ten.
4. Use figures to write the following number: 5 billion, 703 million, 5 thousand.
5. 24 is a common multiple of 4 and 6. What is the least common multiple of 4 and 6?
What is a common multiple of 3, 5, and 6? What is the least common multiple?
6. If 72813654 is written as 72 81 36 54, explain why it is easy to tell that this number is divisible by the digit 9.
7. Read Chapter 10, Section 15, Examples 4 through 10, Page 136.
8. Read the numbers: 48,530,000; 39,037,602,385; 9,402,061.
9. Write these numbers: 12, 379, 500, 333, 2,000, 1,250,794.
10. Find the average of 150, 245, 410, and 1,296.
11. Arrange the numbers in descending order: 12, 10, 2, 1, 3, 6, 9, 4, 11, 5, 7, 8.
12. What is 6,671,873 rounded to the nearest hundred thousand: 6,600,000 or 6,700,000?
13. John said, "If 13 and 4 are 17, then 14 and 3 are 17." Is he correct?
14. To solve the problem, do we "add 300," or do we "take away 300"?
15. How much less than 63 is 42? Mary first said, "The answer is 11", and then she said "No, it is 21." Which is correct?
16. How many minutes are there in a 24-hour day?
17. If 1-ary and unary are the same, are 2-ary and binary the same? What is the same as 3-ary?
18. Is the answer 36—or 37—cents a dozen?
19. Is 1971 a 2-, 3-, or 4-place number?
20. Write the largest 3-digit, 4-digit, and 5-digit numbers you know.
21. Name six-, seven-, and eight-sided figures.
22. Tell the meaning of each of the following signs: $+$, $-$, \times , \cdot , \div , \pm , \mp , $+$ —, $-$ +.
23. Add: $1943 + 462 + 92 + 233 + 4 + 78$.
24. If $5 + 10$ equals 15, what does $10 + 5$ equal?
25. Does $5 - 2$ name a whole number? $9 - 6$?
26. Add or subtract as the signs tell you: $6942 + 3819$; $294,107 + 365,904$; $10,259 - 7386$; $362 - 41$.
27. If 100×4 equals 400, what does 100×5 equal? 100×6 ? 100×7 ?

LESSON 2

NUMERALS AND THE NUMERIC INDICATOR (CONTINUED)

§15. Numeric Indicator and Punctuation Marks:

- a. The numeric indicator must be used after a hyphen connecting a numeral to a preceding word or punctuation mark.

- (1) hydrogen-3

hydrogen-3

- (2) 1-to-1 correspondence

1-to-1 correspondence

- (3) He lived from 1885?-1946.

He lived from 1885?-1946.

- b. The numeric indicator must not be used after a hyphen connecting a numeral to a preceding numeral or other mathematical expression, or after the mathematical comma in a long numeral. The numeric indicator must be used in all other instances when a numeral follows a punctuation mark.

- (1) Do exercises 1-9 and 35-45.

Do exercises 1-9 and 35-45.

- (2) The marks ranged from 65-100.

The marks ranged from 65-100.

- (3) 1,674,932

1,674,932

- (4) "12 dozen"

"12 dozen"

- (5) "4 + 4" is another name for '8'.

"4 + 4" is another name for '8'.

- (6) '71

'71

- (7) He arrived at 7:45.

He arrived at 7:45.

- (8) 7:45-8:45

7:45-8:45

- (9) The answer—25—is correct.

- (10) 50—60

§16. Numeric Indicator With the Minus Sign: The numeric indicator must be used before a numeral following a minus sign when the minus sign follows a space or a punctuation mark, or begins a braille line.

- (1) Is -5 a negative number?

- (2) Is $-6 < -1$?

Figure 1 shows four 3x3 dot patterns. Pattern (a) has 6 dots at (1,1), (1,3), (2,2), (3,1), (3,2), and (3,3). Pattern (b) has 8 dots at (1,1), (1,2), (1,3), (2,2), (2,3), (3,1), (3,2), and (3,3). Pattern (c) has 7 dots at (1,1), (1,2), (1,3), (2,1), (2,2), (3,2), and (3,3). Pattern (d) has 10 dots at (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), and (3,4).

- (3) ± 5 means $+5$ and -5 .

- (4) “ -3 is a negative number.”

- (5) -4 is the opposite of $+4$.

- $$(6) \quad -50 + 9 - 2 = -43$$

DECIMAL POINT

Decimal Point

American • • •

Continental ,

§17. Use of the Decimal Point: Although the symbols for the American and Continental decimal points differ in print, the difference is not shown in braille. A transcriber's note should be included at the beginning of the braille text to inform the reader of the Continental usage in the ink-print edition.

In a numeral, no space should be left between the decimal point and the digits to which it applies.

- (1) Is 1.306 less than 1.31?

- (4) The tolerance is $\pm.005$, not $-.005$.

- (5) Is “ $-.55$ ” the correct answer?

MONETARY, PERCENT, AND PRIME SIGNS

§19. Monetary and Percent Signs:

Monetary Signs

Cent	¢	⠠ ¢ ⠠
Dollar	\$	⠠ \$ ⠠
Pound Sterling	£	⠠ £ ⠠
Percent Sign	%	⠠ % ⠠

Monetary and percent signs must be placed in the same position as in ink-print. No space should be left between monetary or percent signs and their related quantities or symbols. Monetary and percent signs are mathematical symbols and must be punctuated accordingly.

- $$(1) \quad 25¢ - 5¢ = 20¢$$

- (2) $.05¢$ is what part of a quarter?

- (3) $\$2.50 + \$2.50 = \$5.00$

- (4) \$.07

- (5) £4

- (6) 60% of $6 = 3.6$

- (7) 35 is what % of 75?

§22. Ellipsis and Long Dash —

Any dot or series of dots in print representing an omitted term, entry, or line is an ellipsis. In braille, the ellipsis must be represented by a minimum of three dots.

When a dash is used to denote an omission in print, the long dash must be used.

§23. Spacing With Ellipsis and Long Dash:

a. The ellipsis and the long dash should generally be preceded and followed by a space. However, no space should be left between an ellipsis or long dash and a related decimal point, dollar, cent, pound sterling, percent, or prime sign.

- (1) Sally, Ann, and . . . are coming.

- (2) John said . .

- (3) Two and — are ten.

- (4) $14.9 - 12.3 = \underline{\hspace{1cm}}.6$

- (5) $7c + 9c = \dots c$

- $$(6) \quad \dots c + 16c = 30c$$

A 2D grid of 10x10 points. The points are arranged in a regular grid. 15 points are highlighted in black. The highlighted points are at (1,1), (1,4), (1,7), (2,2), (2,5), (2,8), (3,3), (3,6), (3,9), (4,4), (4,7), (5,5), (6,6), (7,7), and (8,8). The other 65 points are white.

- (7) $\$25.00 + \$25.00 = \$\text{---}$

- (8) $.004 = \underline{\hspace{1cm}}\%$

- (9) \$7.35 = £_____

- (10) $24'' = \underline{\hspace{1cm}}'$

b. A space must be left between a sign of operation and an ellipsis or long dash.

- (1) $5 \times \dots = 15$

(2) $2 + 4 + 16 + 32 + \dots + 712$



(3) $17 \times \text{---} = 51$



(4) $43 \text{---} \text{---} = 27$

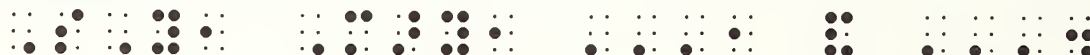


(5) $\text{---} - 15 = 8$



§24. **Punctuation With the Ellipsis and Long Dash:** The ellipsis and the long dash must be punctuated in accordance with their context—literary punctuation in literary context and mathematical punctuation in mathematical context. When the nature of the text is in doubt, the ellipsis and the long dash must be punctuated mathematically. Except for the hyphen, no space should be left between an ellipsis or a long dash and a related punctuation mark.

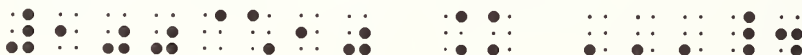
(1) Sally, Mary, ..., and



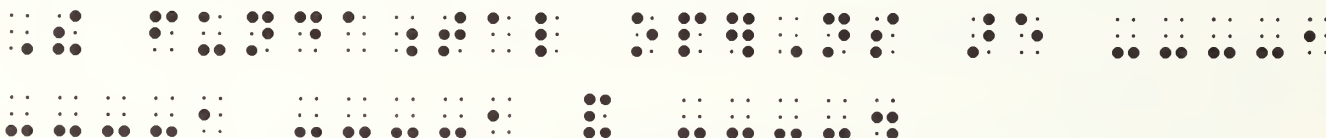
(2) 2, 4, 6, ..., 10.



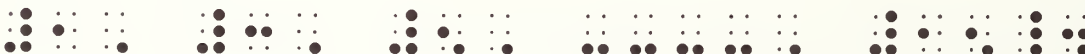
(3) $100 \times 10 = \dots$



(4) The fundamental operations are —, —, —, and —.



(5) 1, 3, 5, —, 15.



(6) $9 \times 12 = \text{---}$.



(7) It is a —-sided figure.



§25. **General Omission Symbol:**

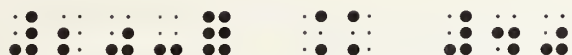


a. The general omission symbol must be used to replace an omission in print represented by a blank space or by a question mark standing alone or in combination with hyphens or a dash. The general omission symbol must be spaced as the material which it replaces, and punctuated mathematically.

(1) $50 \div 10 =$



(2) $20 - = 40$



(3) $6 + ? = 15$



(4) $? - 10 = 30$

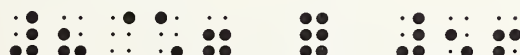


(5) $24 ? 12 = 2$



(the question mark represents a sign of operation)

(6) $8 \times 7 \text{ ? } 56$



(the question mark over a dash represents a sign of comparison)

(7) $42 \times 3 = -?-$



(8) $10 \text{ is } ?\% \text{ of } 100$



(9) $27\phi + 19\phi = -?- \phi$



(10) $5, 10, ?, 20, ?.$

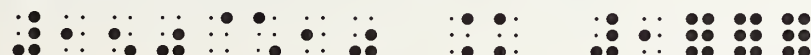


b. The number of general omission symbols to be used in braille must correspond to the number of omission signs used in print.

(1) $150 \times 10 = ????$



(2) $150 \times 10 = 1???$



(3) $150 \times ?? = 1500$



PLURAL AND POSSESSIVE ENDINGS

§26. Apostrophe-s: 's



a. When apostrophe-s is used to form the plural or possessive of a mathematical expression, the punctuation indicator must be used before the apostrophe. If the apostrophe is omitted in print, it must be omitted in braille.

- (1) 1's and 2's



- (2) How many 10's are there in 50?



- (3) Insert +s or =s to make true sentences.



- (4) 1s and 2s



- (5) There are four 4s in 16.



- (6) +s or =s



b. When apostrophe-s or "s" is attached to a mathematical expression, it becomes part of that expression and must be punctuated mathematically.

- (1) 1's, 2's, and 3's.



- (2) 1s, 2s, and 3s.



ORDINAL ENDINGS

§27. **Ordinal Endings With Mathematical Expressions:** Ordinal endings are formed by attaching "st," "nd," "rd," or "th" to a numeral or other mathematical expression. If the "n" or the "r" is omitted from an ordinal ending in print, it must be omitted in braille. The contractions for "st" and "th" must not be used. Ordinal endings are part of a mathematical expression and must be punctuated mathematically.

- (1) 1st, 2nd, 3rd, and 4th.



(2) 1st, 2d, and 3d.



FORMAT (CONTINUED)

§28. **Margins for Instructions Preceding Itemized Material:** When itemized material is preceded by instructions, the instructions must begin in cell 5, and runovers must begin in cell 3. One line must be left blank above such instructions unless they begin a braille page or follow a new page-separation line. A line must not be left blank below the instructions. The last line of an instruction and the first line of the related problem must be on the same braille page.

(1)

53

Tell whether the following ratios
are equivalent.

1. $3 : 2 = 75 : 50$

2. $6 : 4 = 15 : 30$

Which of the following sentences
are true? Which are false?

3. $328 \div 4 = 41 \times 2$

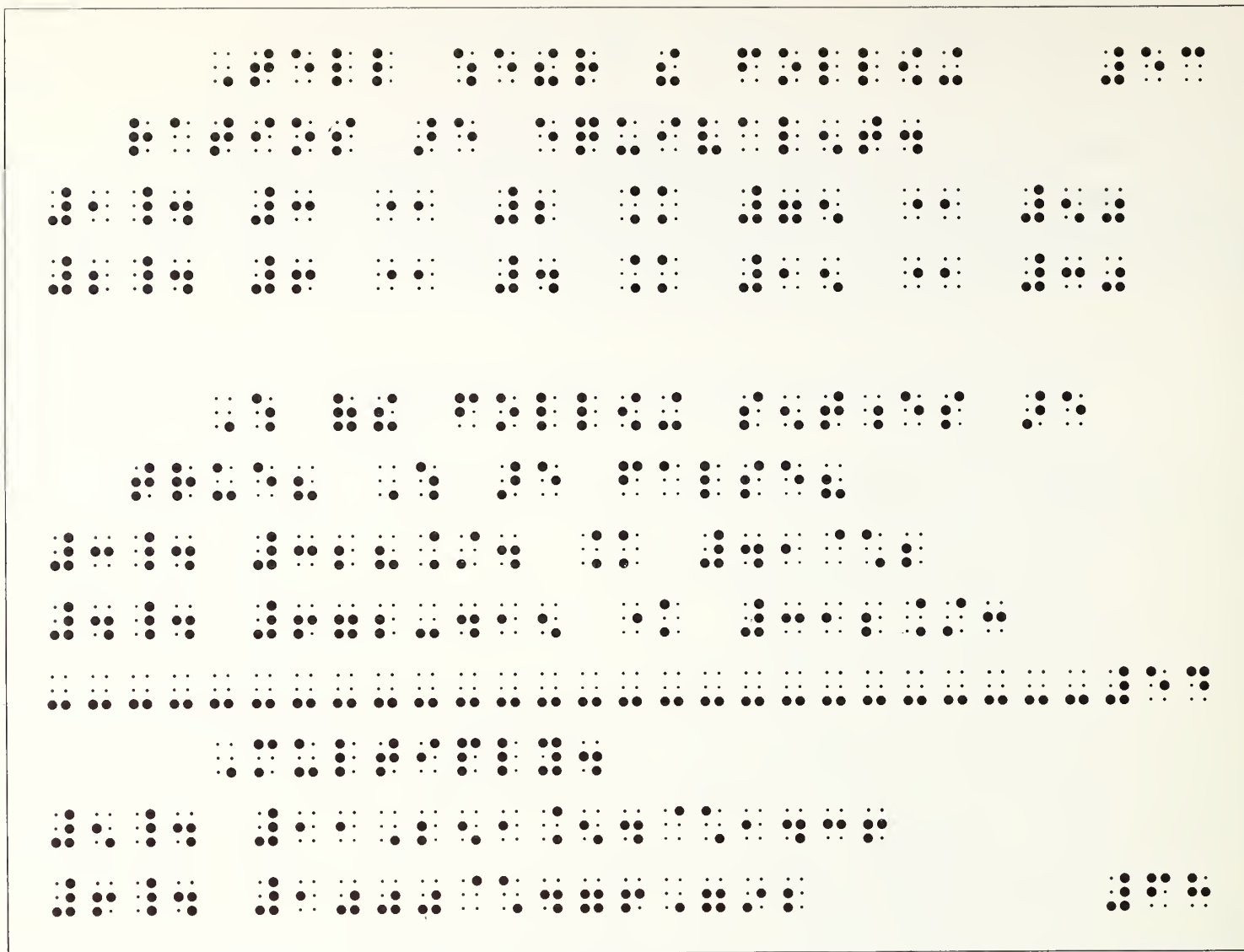
4. $672 - 415 < 312 \div 3$

54

Multiply.

5. $11,251.54 \times 1436$

6. $1000 \times 476,792$



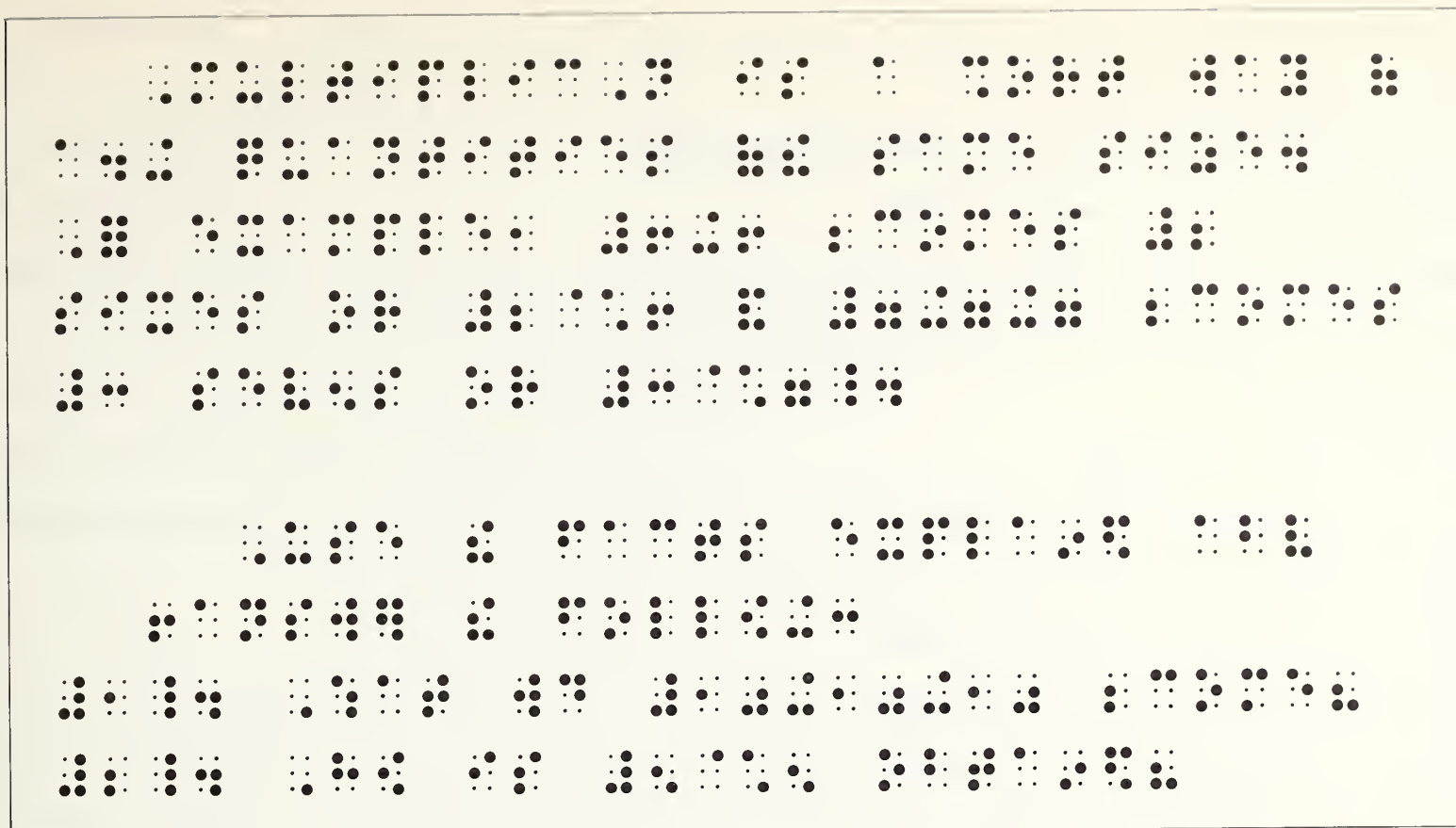
§29. **Margins for Unitemized Explanatory Portions of Text:** As in English braille, paragraphs in explanatory portions of text must begin in cell 3, and all runovers must begin in cell 1.

(1)

Multiplication is a short way of adding quantities of the same size. For example, $6 + 6$ becomes 2 sixes or 2×6 and $7 + 7 + 7$ becomes 3 sevens or 3×7 .

Use the facts explained above to answer the following:

1. What would $10 + 10 + 10$ become?
2. How is 5×5 obtained?



HOMEWORK

Prepare the following homework for submission to your teacher. Proofread carefully.

EXERCISE 2

1. Is there a 1-to-1 correspondence between the members of the sets in Exercise 2?
2. The odds are 10-to-1 against winning. What does this mean in terms of dollars?
3. Did Plato live from 427?-347 or from 428?-348?
4. From the table, tell which scores are included in the intervals 0-9, 15-19, and 40-49.
5. Find the total precipitation for the years 1954-56 and 1957-58.
6. Think: "9 and 9 are 18," and "2 more are 20."
7. In 7,462, does the "7" mean 7, or does it mean 7000? What does the "4" mean? the "6"? the "2"?
8. Is "30 + 5" another name for 35?
9. Are '7 • 8' and '8 • 7' names for the same number?
10. In 1948, '49, and '50, the average rainfall was 24.0, 23.1, and 64.0. What was the total rainfall for the 3 years?
11. Find the number of minutes between 10:55 and 11:10; 10:55 and 11:45.

12. From the data, what is the difference in growth between the 5—10 and the 15—20-year groups?
13. Which is the average—62 or 63?
14. Bob practiced from 9:45-11:30. How many minutes did he practice?
15. Is the product of -1 and -3 a negative number?
16. What is the sum of -119 and -67 ?
17. Find the integer named by: $-24-0$; $-5-0$; $-40-0$.
18. Does $-45 \div 9 = -5$?
19. What is the sum of -10 and $+8$?
20. A proper divisor of -42 is ± 2 . Is ∓ 2 a proper divisor?
21. Which arrows represent “ -5 ” and “ -10 ” on the number line?
22. What is the value of 0.02 ? of 0.002 ? of 0.0025 ?
23. Round to the nearest tenth: 9.45, 3.86, 423.55, 2.981, 9.815.
24. Round .469 to the nearest hundredth. Is your answer .46, or is it .47?
25. Add: $.236 + 1.49 + .01 + 7.6$.
26. Are the following sentences true or false: $.04 > .039$; $.305 < .34$?
27. Does $.092 + .171 + .355 = .618$?
28. What is “ $.6 \times 7.25$ ”? Would you have to multiply again to find “ $7.25 \times .6$ ”?
Does $4.350 \div .6 = 7.25$? Does $4.350 \div 7.25 = .6$?
29. List the tenths from .1-to-.10.
30. Name the hundredths from .01-.25.
31. Find the sum of $-.38$ and $-.42$.
32. Which point is farther from zero: $-.5$ or -5 ?
33. The maximum error is $\pm .05$, not $-.05$.
34. Find the point labeled “ $-.75$.”
35. In one week Beth earned \$2.75, 60¢, and 85¢. She spent 25¢, 58¢, and \$1.32. How much did she have left?
36. The sign read “Candy bars—6 for 29¢.” Jim bought 12 candy bars. How much change did he receive from \$1.00?
37. Find: $\$9.86 + \$.07 + \$468.57$.
38. What does $\$.75 - \$.52$ equal?
39. Does $41¢ - 32¢ = 9¢$? Can 9¢ also be written \$.09?

40. What is 50% of \$120?
41. Does $5\% + 62\% = 67\%$, or $.67\%$?
42. 55 is what % of 100?
43. Is 6' the same as 72"?
44. Does $5'7'' + 2'9'' = 8'4''$?
45. If the total length of a rectangle is 4", 23% of it is $.23 \times 4''$, or $.9''$, and 40% of it is $.40 \times 4''$, or $1.6''$.
46. Carl, Carol, and ... are part of the set. Bob and ... are not.
47. Explain the meaning of the dots in 1.141222 ... 2 ...

Write the missing information that will make each sentence true.

48. $16 + 420 = \dots$
49. $12 \times \dots = 144$
50. $1 + 3 + \dots + \dots + 81 + 243 = 364$
51. Ten and _____ are twenty.
52. 100 times _____ is 3400.
53. $146 - \underline{\hspace{1cm}} = 32$
54. $25\phi + 50\phi = \dots\phi$
55. $73\phi - \dots\phi = 24\phi$
56. $\$1.28 + 4.39 + 6.75 = \$\underline{\hspace{1cm}}$
57. $40.23 \div 23 = \underline{\hspace{1cm}}.75$
58. $25 = \underline{\hspace{1cm}}\%$ of 100
59. $36'' = \underline{\hspace{1cm}}'$
60. The opposite of multiplication is ..., and the opposite of addition is
61. Fifteen plus twelve are _____, and six more are _____.
62. 1, 3, 5, ..., 9,
63. 2, 4, _____, 8, _____.
64. $.432 - .0087 =$
65. $8 : 15 :: 24 : ?$
66. $51,858 \div ? = 402$
67. $? + 64 + 58 + 97 = 265$
68. $36,000,000 = 3.6 \times \text{-?}$
69. 25 is ? % of 50
70. $1435 \times 6 = 86??$

71. .42, .43, .44, ?, ?, ?

Use +, —, \times , or \div to make true sentences.

72. $100 \text{ ? } 35 = 3500$

73. $650 \text{ ? } 274 = 376$

74. $1,354 \text{ ? } 52 = 1,406$

75. $70.45 \text{ ? } 14.09 = 5$

76. $.006 \text{ ? } .002 = .003$

Replace each ? by the correct sign: =, <, or >.

77. $8174 + 9698 \text{ ? } 18,872$

78. $-10 \text{ ? } -8$

79. $12,589 \text{ ? } 589 + 11,000$

In our work so far we have learned about numbers, equalities, inequalities, monetary computation, and omissions. We will now study plural, possessive, and ordinal forms of numbers. Remember, these forms can also be used with other mathematical symbols or expressions.

80. Mathematical sentences use signs such as +'s or ='s.

81. Which is more: two 3's, or three 2's?

82. What is $10 \div 2$? Are there 5 2s in 10? Are there 2 5s in 10?

83. Find the 9th term of $1 + 4 + 7 + 10 + \dots$

84. Find the sum of the 1st seven terms of the sequence 3, —6, 12, \dots

85. Beginning with 5, find the 23d odd integer.

86. Which terms in the sequence above are 1st, 2nd, and 3rd?

LESSON 3

ALPHABETS

Specific provision is made for the transcription of the letters of the English (Roman), German, Greek, Hebrew, and Russian (Cyrillic) alphabets. The letters of each alphabet and their braille equivalents are listed on the following pages. Note that the letters of the Hebrew alphabet have no capitalized form.

English (Roman) Alphabet

Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Sanserif capitalized	Braille equivalent	Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Sanserif capitalized	Braille equivalent
a	A	<i>a</i>	<i>À</i>	A	⠁	n	N	<i>n</i>	<i>ℕ</i>	N	⠎
b	B	<i>b</i>	<i>ℬ</i>	B	⠃	o	O	<i>o</i>	<i>ℴ</i>	O	⠏
c	C	<i>c</i>	<i>℄</i>	C	⠉	p	P	<i>p</i>	<i>℘</i>	P	⠑
d	D	<i>d</i>	<i>ℊ</i>	D	⠙	q	Q	<i>q</i>	<i>ℚ</i>	Q	⠒
e	E	<i>e</i>	<i>ℰ</i>	E	⠑	r	R	<i>r</i>	<i>℞</i>	R	⠓
f	F	<i>f</i>	<i>ℱ</i>	F	⠋	s	S	<i>s</i>	<i>™</i>	S	⠔
g	G	<i>g</i>	<i>ℊ</i>	G	⠗	t	T	<i>t</i>	<i>™</i>	T	⠞
h	H	<i>h</i>	<i>ℋ</i>	H	⠈	u	U	<i>u</i>	<i>℣</i>	U	⠤
i	I	<i>i</i>	<i>ℐ</i>	I	⠊	v	V	<i>v</i>	<i>ℤ</i>	V	⠥
j	J	<i>j</i>	<i>ℐ</i>	J	⠛	w	W	<i>w</i>	<i>Ⅎ</i>	W	⠦
k	K	<i>k</i>	<i>℔</i>	K	⠅	x	X	<i>x</i>	<i>Ⅎ</i>	X	⠭
l	L	<i>l</i>	<i>ℒ</i>	L	⠙	y	Y	<i>y</i>	<i>℣</i>	Y	⠽
m	M	<i>m</i>	<i>™</i>	M	⠍	z	Z	<i>z</i>	<i>ℤ</i>	Z	⠵

German Alphabet

Name of letter	Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Braille equivalent
ah	a	A	<i>a</i>	<i>A</i>	⠁
beh	b	B	<i>b</i>	<i>B</i>	⠃
tseh	c	C	<i>c</i>	<i>C</i>	⠉
deh	d	D	<i>d</i>	<i>D</i>	⠙
eh	e	E	<i>e</i>	<i>E</i>	⠑
eff	f	F	<i>f</i>	<i>F</i>	⠋
gheh	g	G	<i>g</i>	<i>G</i>	⠗
hah	h	H	<i>h</i>	<i>H</i> or <i>G</i>	⠈
ee	i	I	<i>i</i>	<i>I</i>	⠊
yaht	j	J	<i>j</i>	<i>J</i> or <i>I</i>	⠞
kah	k	K	<i>k</i>	<i>K</i>	⠅
ell	l	L	<i>l</i>	<i>L</i>	⠙
em	m	M	<i>m</i>	<i>M</i>	⠓

Name of letter	Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Braille equivalent
en	n	N	<i>n</i>	<i>N</i>	⠝
oh	o	O	<i>o</i>	<i>O</i>	⠕
peh	p	P	<i>p</i>	<i>P</i>	⠏
koo	q	Q	<i>q</i>	<i>Q</i>	⠒
err	r	R	<i>r</i>	<i>R</i>	⠗
ess	s	S	<i>s</i> or <i>p</i>	<i>S</i>	⠎
teh	t	T	<i>t</i>	<i>T</i>	⠞
oo	u	U	<i>u</i>	<i>U</i>	⠥
fao	v	V	<i>v</i>	<i>V</i>	⠺
veh	w	W	<i>w</i>	<i>W</i>	⠺
iks	x	X	<i>x</i>	<i>X</i>	⠭
ypsilon	y	Y	<i>y</i>	<i>Y</i> or <i>Y</i>	⠽
tset	z	Z	<i>z</i>	<i>Z</i>	⠵

Greek Alphabet (Standard)

Name of letter	Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Braille equivalent	Name of letter	Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Braille equivalent
alpha	α	Α	α	Α	⠠⠠	omicron	ο	Ο	ο	Ο	⠠⠠
beta	β	Β	β	Β	⠠⠠	pi	π	Π	π	Π	⠠⠠
gamma	γ	Γ	γ	Γ	⠠⠠	rho	ρ	Ρ	ρ	Ρ	⠠⠠
delta	δ	Δ	δ	Δ	⠠⠠	sigma	σ	Σ	σ	Σ	⠠⠠
epsilon	ε	Ε	ε	Ε	⠠⠠	tau	τ	Τ	τ	Τ	⠠⠠
zeta	ζ	Ζ	ζ	Ζ	⠠⠠	upsilon	υ	Υ	υ	Υ	⠠⠠
eta	η	Η	η	Η	⠠⠠	phi	φ	Φ	φ	Φ	⠠⠠
theta	θ	Θ	θ	Θ	⠠⠠	chi	χ	Χ	χ	Χ	⠠⠠
iota	ι	Ι	ι	Ι	⠠⠠	psi	ψ	Ψ	ψ	Ψ	⠠⠠
kappa	κ	Κ	κ	Κ	⠠⠠	omega	ω	Ω	ω	Ω	⠠⠠
lambda	λ	Λ	λ	Λ	⠠⠠	sampi	ϝ				⠠⠠
mu	μ	Μ	μ	Μ	⠠⠠	stigma	Ϛ				⠠⠠
nu	ν	Ν	ν	Ν	⠠⠠	vau	Ϝ				⠠⠠
xi	ξ	Ξ	ξ	Ξ	⠠⠠	koph (goph)	Ϟ or ϟ				⠠⠠

Hebrew Alphabet

Hebrew letters do not possess a capitalized form.

Name of letter	Regular	Script	Braille equivalent	Name of letter	Regular	Script	Braille equivalent
aleph	א	א	⠠	lamed	ל	ל	⠠
veth	ב	ב	⠠	mem	מ	מ	⠠
gimel	ג	ג	⠠	nun	נ	נ	⠠
daleth	ד	ד	⠠	samekh	ס	ס	⠠
heh	ה	ה	⠠	ayin	ע	ע	⠠
vav	ו	ו	⠠	feh	פ	פ	⠠
zayin	ז	ז	⠠	tsadi	צ	צ	⠠
cheth	ח	ח	⠠	koph	ק	ק	⠠
teth	ט	ט	⠠	resh	ר	ר	⠠
yod	י	י	⠠	sin	ש	ש	⠠
chaph	כ	כ	⠠	thav	ת	ת	⠠

Russian (Cyrillic) Alphabet

Name of letter	Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Braille equivalent	Name of letter	Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Braille equivalent
ah	а	А	а	А	⠠	zeh	з	З	з or ж	З	⠠
beh	б	Б	б	Б	⠠	ee	и	И	и	И	⠠
veh	в	В	в	В	⠠	kah	к	К	к or н	К	⠠
gheh	г	Г	г	Г	⠠	ell	л	Л	л	Л	⠠
deh	д	Д	д or 2	Д	⠠	em	м	М	м	М	⠠
yeh	е	Е	е	Е	⠠	en	н	Н	н	Н	⠠
zheh	ж	Ж	ж or ж	Ж	⠠	oh	о	О	о	О	⠠

Russian (Cyrillic) Alphabet (continued)

Name of letter	Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Braille equivalent	Name of letter	Regular uncapitalized	Regular capitalized	Script uncapitalized	Script capitalized	Braille equivalent
peh	п	П	<i>n</i>	<i>П</i>	⠠⠠⠠	ch eh	ч	Ч	<i>ʃ</i>	<i>Ч</i>	⠠⠠⠠
err	р	Р	<i>ʀ</i>	<i>Р</i>	⠠⠠⠠	sh ah	ш	Ш	<i>ʃ</i> or <i>ʃ</i>	<i>Ш</i>	⠠⠠⠠
ess	с	С	<i>c</i>	<i>С</i>	⠠⠠⠠	sh ch ah	щ	Щ	<i>ʃ</i>	<i>Щ</i>	⠠⠠⠠
teh	т	Т	<i>m</i> or <i>m̄</i>	<i>Т</i>	⠠⠠⠠	yer ih	ы	Ы	<i>ʃ</i>		⠠⠠⠠
oo	у	У	<i>y</i>	<i>У</i>	⠠⠠⠠	eh	э	Э	<i>ə</i>	<i>Э</i>	⠠⠠⠠
eff	ф	Ф	<i>ʃ</i>	<i>Ф</i>	⠠⠠⠠	yu	ю	Ю	<i>ʃ</i>	<i>Ю</i>	⠠⠠⠠
kh ah	х	Х	<i>x</i>	<i>Х</i>	⠠⠠⠠	yah	я	Я	<i>ʃ</i>	<i>Я</i>	⠠⠠⠠
tseh	ц	Ц	<i>ʃ</i>	<i>Ц</i>	⠠⠠⠠						

GERMAN, GREEK, HEBREW, AND RUSSIAN ALPHABETS
AND ALPHABETIC INDICATORS

Alphabetic Indicators

German Letter Indicator	⠠⠠⠠
Greek Letter Indicator	
For Standard Letters	⠠⠠⠠
For Alternative Forms of Letters	⠠⠠⠠ ⠠⠠⠠
Hebrew Letter Indicator	⠠⠠⠠ ⠠⠠⠠
Russian (Cyrillic) Letter Indicator	⠠⠠⠠ ⠠⠠⠠
Capitalization Indicator (single)	⠠⠠⠠

§30. Use of German, Greek, Hebrew, and Russian Alphabetic Indicators:

a. The appropriate alphabetic indicator for German, Greek, Hebrew, and Russian letters *must always be* used to identify the alphabet to which the letter belongs. The effect of an alphabetic indicator extends only to the letter which follows it. Thus, in a sequence of unspaced letters, the appropriate alphabetic indicator must be used before each letter.

(1) a

(German uncapitalized ah)

(2) α

(Greek uncapitalized alpha)

(3) \mathcal{N}

(Hebrew alef)

(4) a

(Russian uncapitalized ah)

(5) $\alpha\beta\gamma$

(Greek uncapitalized alpha, beta, gamma)

(6) $\alpha \in \mathfrak{b} \not\sim \mathfrak{b}$

(Greek uncapitalized alpha, German uncapitalized beh, Hebrew alef, Russian uncapitalized beh)

$$(7) \quad \alpha + \beta = \pi - \theta$$

(8) $\mu \pm 1.645 \sigma$

$$(9) \quad \pi < 0 < 2\pi$$
$$(10) \quad \mathbf{a}_\alpha + \mathbf{b}_\beta$$

§31. **Alternative Forms of Greek Letters:** The following Greek letters possess an alternative uncapitalized print form. The difference is shown in braille by placing the Greek letter alternative form indicator $\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$ in front of these letters. If a Greek letter is represented by its alternative form *instead of* its standard form throughout the print text, the symbol for the standard form must be used in braille. However, a transcriber's note must be included at the beginning of the text to inform the reader of such changes in braille usage. The alternative form should be used in braille only when both forms—standard and alternative—appear in the same print text.

Name of letter	Alternative print form	Braille equivalent
alpha	α	α
beta	β	β
theta	ϑ	ϑ
sigma	ς	ς
phi	φ	φ

- (1) \emptyset and ϕ have separate identities.

- (2) $\emptyset \in \mathcal{V}$ and $\emptyset \in \Theta$ are the least elements of each set.

ENGLISH ALPHABET AND THE ENGLISH LETTER INDICATOR

English Letter Indicator

Capitalization Indicator (single) ::

§32. Use of the English Letter Indicator With English Letters in Regular Type:

a. The English letter indicator must be used with a single capitalized or uncapitalized English letter or with an uncapitalized combination of letters corresponding to a short-form word of English braille *if, in braille, the letter or short-form combination is preceded by a space or by one or more punctuation marks and is followed by a space or by one or more punctuation marks.*

When the English letter indicator is used, its effect extends to the letter or to the entire short-form combination following it. If a letter is capitalized, the capitalization indicator must be placed between the English letter indicator and the letter.

Single letters and short-form combinations which are not words or abbreviations are mathematical expressions and must be punctuated accordingly.

- (1) Set R contains x elements.



- (2) Vitamins A and D.



- (3) p is larger than q .



- (4) cd is more than ac .



- (5) l, m, n are in Set S.



- (6) a, b, c, . . . , z.



- (7) Find A, B, and C.



- (8) ab , cd , and gd are equal.



- (9) $n:v$ means “the mapping n of v .”



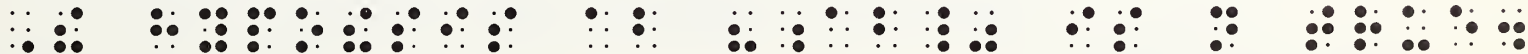
- (10) "v varies as x."



- (11) "a" is less than "B".



- (12) The hypothesis about “ab” is not true.



- (13) A. $12 + 6 - 8 = ?$

B. $4 \times 6 \times 1000 = ?$



Since the effect of the capitalization indicator extends only to the following letter, each capitalized letter in a mathematical sequence of letters must be capitalized individually. Such sequences of letters are mathematical expressions and must be punctuated accordingly.

- (1) Is ca the same as ac?

Is ca the same as ac?

- (2) Measure line wh.

Measure line wh.

- (3) "xy is the same as yx."

"xy is the same as yx."

- (4) Find the points on the xy- and st-axes.

Find the points on the xy- and st-axes.

- (5) Prove PQRS is a rhombus.

Prove PQRS is a rhombus.

- (6) Find chords AC, AL, and EF.

Find chords AC, AL, and EF.

- (7) If th = ef, then lm = ch.

If th = ef, then lm = ch.

d. The English letter indicator must not be used with one or more English letters in regular type which occur in an unspaced sequence of terms consisting of a mixture of numerals, letters from other alphabets, signs of operation, or any other unspaced mathematical symbol.

- (1) $3a \times 4b = 12ab$

$3a \times 4b = 12ab$

- (2) $2x$ means $2 \times x$.

$2x$ means $2 \times x$.

- (3) Δy is an increment in y .

Δy is an increment in y .

- (4) $C = 2\pi r + \pi \Delta r$

$C = 2\pi r + \pi \Delta r$

- (5) $s = r\theta$

$s = r\theta$

(6) $a + b = c$



(7) $e \times e \times e = e\text{-cubed}$



(8) $Ax + By + Cz + D = 0$



(9) $CE + ED > DC$



(10) $ab \times cd = ac \times bd$



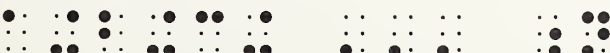
(11) Substitute $-y$ for $+y$.



(12) $a + a + a + a + \dots + a$



(13) $a + b + c + \dots n$



(14) Sides d' and d are similar.



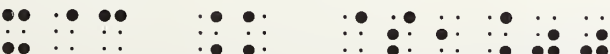
(15) $R'S'T'$



(16) 35 equals $N\%$ of 120



(17) $x\phi = \$1.00$



(18) 40 dimes equals $\$x$



e. When a single English letter or a combination of unspaced letters has a plural, possessive, or ordinal ending, the English letter indicator must be used or must not be used as though such endings were not present. The English letter indicator must not be used with the letter "s" when the "s" is part of the apostrophe-s combination.

- (1) a's, b's, and c's.

⠠⠁⠛⠛⠠⠃⠛⠛⠠⠉⠛⠛

- (2) A's, B's, and C's.

⠠⠠⠁⠛⠛⠠⠠⠃⠛⠛⠠⠠⠉⠛⠛

- (3) ps, qs, and rs.

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- (4) Ps, Qs, and Rs.

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- (5) The ab's, cd's, and ef's.

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- (6) The ABC's and XYZ's.

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- (7) Find the nth root of x.

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- (8) The abth and jkth columns.

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- (9) 2nth

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

ABBREVIATIONS

§34. **Capitalization and Punctuation With Abbreviations:** Abbreviations must be capitalized and punctuated in accordance with the rules of English braille. The single capitalization indicator ⠠ must be used before a single capitalized letter, and the double capitalization indicator ⠠⠠ must be used before a sequence of two or more unspaced capitalized letters when each letter represents an individual word. The effect of the double capitalization indicator is terminated by any symbol other than a letter.

§35. Definition of Abbreviations:

- a. Abbreviations consist of the following items:

- i. Universal literary abbreviations commonly listed in a dictionary.

- (1) Mon.

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(2) Washington, D.C.



(3) U.S.A.



v. Initials of agencies, organizations, business firms, etc.

(1) CIO



(2) B.P.O.E.



(3) IBM



vi. Abbreviations formed from the principal letters of a word, phrase, or name.

(1) ans.



(2) Atty. Gen.



(3) Geo.



vii. Special abbreviations confined to a special field or a particular book.

(1) SAS means “side angle side”.



(2) iff means “if and only if”.



b. A single letter or sequence of letters which does not represent a word or a phrase, as well as model numbers, serial numbers, etc., must not be considered abbreviations and must be transcribed according to other rules of the code. When there is doubt whether a particular item is an abbreviation, it must be treated as if it were not an abbreviation.

(1) Model number 1074FE.



(6) How many min. are there in one hr.?

How many min. are there in one hr.?

(7) statamp-oersted

statamp-oersted

(8) FORTRAN is a computer language.

FORTRAN is a computer language.

(9) Wed., Thurs.

Wed., Thurs.

(10) Ariz., Ark., and Conn.

Ariz., Ark., and Conn.

(11) Read chap. 4.

Read chap. 4.

§38. Abbreviations and the English Letter Indicator:

a. The English letter indicator must be used with an abbreviation consisting of a single letter or a combination of letters corresponding to a short-form word when the abbreviation has no period in print, or, if there is a period, the period merely ends a sentence.

(1) F is the abbreviation for Fahrenheit; the abbreviation for Centigrade is C.

F is the abbreviation for Fahrenheit; the abbreviation for Centigrade is C.

(2) N 30 degrees W.

N 30 degrees W.

(3) 10 g + 10 g = 20 g.

10 g + 10 g = 20 g.

(4) 1 l = 1000 cubic centimeters

1 l = 1000 cubic centimeters

(5) Yr is the abbreviation for "year."

Yr is the abbreviation for "year."

(6) 1 yr = 12 months

1 yr = 12 months

(7) It is a 1-g weight.

It is a 1-g weight.

(8) 1 g-mole

1 g-mole

(9) I-O means "Input-Output".

I-O means "Input-Output".

(10) "d-c" stands for "direct current."

"d-c" stands for "direct current."

(11) 1 light-yr

1 light-yr

b. The English letter indicator must not be used with an abbreviation consisting of a single letter or a combination of letters corresponding to a short-form word if the abbreviation is followed by its related period. When it is doubtful whether a period ends a sentence or applies to an abbreviation, the period must be considered as applying to the abbreviation, and the English letter indicator must not be used.

(1) C. is the abbreviation for Celsius.

C. is the abbreviation for Celsius.

(2) We met Mister P. and Mister Q.

We met Mister P. and Mister Q.

(3) Cubic meters is abbreviated cu. m.

Cubic meters is abbreviated cu. m.

(4) N. 30 degrees W.

N. 30 degrees W.

(5) 10 g. + 10 g. = 20 g.

10 g. + 10 g. = 20 g.

(6) Use a 1-g. weight.

Use a 1-g. weight.

(7) He is 1 yr. old today.

He is 1 yr. old today.

(8) 1 yr. = 12 months

1 yr. = 12 months

- (9) The abbreviation for Centigrade is C.

- (10) 1 light-yr.

c. The English letter indicator must not be used with an abbreviation that consists of two or more letters and does not correspond to a short-form word, whether or not it is followed by a period.

- (1) He worked for 3 hrs.

- (2) 62 cg 8 mg

- (3) Does $1 \text{ km} = 1000 \text{ m}$?

- (4) 7 da = 1 wk

§39. **Abbreviations and the Numeric Indicator:** The numeric indicator must be used before a numeral or a decimal point and its numeral connected to a preceding abbreviation by a hyphen.

- (1) The airplane is a DC-6.

- (2) U-238 means Uranium-238.

- (3) e.g. - .07

The sequence of dot patterns is as follows:

- Pattern 1: Top row (1 dot), middle row (1 dot), bottom row (1 dot).
- Pattern 2: Top row (1 dot), middle row (2 dots), bottom row (1 dot).
- Pattern 3: Top row (2 dots), middle row (2 dots), bottom row (1 dot).
- Pattern 4: Top row (1 dot), middle row (2 dots), bottom row (2 dots).
- Pattern 5: Top row (1 dot), middle row (1 dot), bottom row (2 dots).
- Pattern 6: Top row (2 dots), middle row (2 dots), bottom row (2 dots).
- Pattern 7: Top row (1 dot), middle row (1 dot), bottom row (3 dots).
- Pattern 8: Top row (1 dot), middle row (2 dots), bottom row (3 dots).
- Pattern 9: Top row (2 dots), middle row (2 dots), bottom row (3 dots).
- Pattern 10: Top row (1 dot), middle row (3 dots), bottom row (3 dots).

HOMEWORK

Prepare the following homework for submission to your teacher. Note that letters are used as main outline designations in the last group of itemized problems. Use the margin format principles as stated in §13. Proofread carefully.

EXERCISE 3

Practice writing these German letters:

1. a, b, b, m, r, s, t.

2. A, B, D, M, N, G, I.

3. Prove that a model of \mathfrak{T} is a model of the set of theorems of \mathfrak{T} .

4. Theorem 4.4 shows \mathfrak{D} is an infinite model of \mathfrak{M} and that \mathfrak{D}' is a model of \mathfrak{M} having a cardinality \aleph .

Practice writing the standard and alternate forms of these Greek letters:

5. $\alpha, \beta, \gamma, \delta, \theta, \mu, \pi, \sigma, \phi, \omega$.
6. $\Gamma, \Delta, \Pi, \Sigma, \Phi, \Omega$.
7. $\propto, \delta, \vartheta, \varsigma, \varphi$.
8. Π and Σ denote the product and sum of consecutive terms.
9. Prove that ϕ is the inverse of φ .
10. Construct a circle of radius 1 and obtain the functions of θ , given that: $\theta = 1; \theta = \pi; \theta = \pi + 0.3$.
11. Use the diagram to find the values of α, β , and γ . Does $\alpha + \beta = \gamma$?
12. Prove that for ordinal numbers α, β , and γ , $\gamma\alpha = \gamma\beta$ and $\gamma > 0$ imply $\alpha = \beta$.

Practice writing these Hebrew letters:

13. $\aleph, \beth, \aleph, \beth$.
14. The \aleph is equal to our digit 1.
15. Establish the relation $\aleph \aleph = \aleph$.

Practice writing these Russian letters:

16. $\mathfrak{a}, \mathfrak{b}, \mathfrak{n}, \mathfrak{k}, \mathfrak{y}$.
17. $\mathfrak{A}, \mathfrak{B}, \mathfrak{H}, \mathfrak{K}, \mathfrak{Y}$.
18. Explain why \mathfrak{D}' and \mathfrak{H}' are equal.

Practice writing letters from the English alphabet.

19. There are n elements in Set R .
20. Use the data to prove that r is greater than s .
21. Is line ab equal to line cd ?
22. Study the diagram. Are ac and al equal distances from td ?
23. List the 24 permutations that can be made from the 4 letters a, b, c , and d —taken 3 at a time.
24. Is a, b, c, d, \dots, z a finite sequence?
25. Study diagram a and list the coordinates of the points P, Q, R , and S .
26. The notation $T:V$ means “ T is a function whose domain is V .”
27. Verify: If “ p or q ” is true and “ p ” is false, then “ q ” is true.
28. Use symbolism to represent the negation of a disjunction: p or q , is the conjunction of not- p and not- q .

29. If x , y , and z are used to denote the first, second, and third coordinates, we can speak of the x -, y -, and z -coordinates of a point as well as the x -, y -, and z -axes. Do we have to use the letters x , y , and z ? Can we use other letters and have a -, b -, and c -coordinates and axes?
30. Label the ab - and cd -planes.
31. Can we say that a natural generalization of a binary relation is that of an n -ary relation as a set of ordered n -tuples?
32. Write the multiplication facts for table entries A—H.
33. Solve exercises a-d in Section 5-c. Then read Section D-2.
34. If d varies directly as t , and if $d = 8$ when $t = 4$, find d when $t = 5$.
35. $N = 36\%$ of 1200.
36. Prove: If x , y , and u are real numbers such that $x < y$ and $x = u$, then $u < y$.
37. Express x in terms of r , s , and t : $r : s = t : x$.
38. Prove: If $a < b$ and $c < 0$, then $ac > bc$.
39. Use Theorem 6.4 to explain why $n:v = m:p$.
40. From the data above, explain why " p " = " r ".
41. In triangle ABC , D is a point on AB , E is a point on AC and DE is drawn. $AD = 6$, $DB = 4$, $AC = 15$. If DE is parallel to side BC , the length of EC is: 4, 6, 10.
42. Use numerals to prove that ab is the same as ba .
43. Locate the xy - and st - coordinate planes.
44. Solve: $3b - 3 = 37 - 2b$; $r + r + r - 30 = 180$; $8x - 9 + x - 17 = 3x + 4 + 8x - 12$; $m = 5m - 30$.
45. Draw the graphs of $x - 3y = 14$ and $3x + 2y = 20$.
46. The expression $x - y$ has 2 terms: x and $-y$. Name the terms in $a + b - c$.
47. What number does N stand for in $N \times 9 = 45$? in $5 \times N = 45$?
48. $ab + cd - ef$ is a trinomial. Which are its terms: ab , cd , ef , or ab , $+cd$, $-ef$?
49. If C in $C = \pi d$ is 100, $d = \underline{\hspace{1cm}}$.
50. Write the logarithmic equation for $S = 2\pi rh$ and $C = \pi d$.
51. What does the notation Δx and Δy stand for?
52. If triangles DEF and $D'E'F'$ are similar, is it true that each pair of angles D and D' , E and E' , C and C' are equal?
53. Bob invested " d " dollars, part at $x\%$ and part at $y\%$. If his annual income is " I " dollars, what amount did he invest at $x\%$?
54. The total of the nine letters in the word "addressed" contains 3 d's, 2 e's, and 2 s's. How many permutations are possible with the letters taken all together?
55. Delineate the ab 's, cd 's, and xy 's solutions.

56. Use symbols to write the n th root of a .
57. Write the expression which denotes the p th power of the non-negative q th root of b when $b > 0$.
58. What represents the cell corresponding to the ab th row and jk th column?

Solve the following problems.

- [illegible]

LESSON 4

BOLDFACE, ITALIC, SCRIPT, AND SANSERIF TYPE

§40. **Type Forms and Type-Form Indicators:** Specific provision is made by the use of type-form indicators for the transcription of boldface, italic, script, and sanserif print type forms. A type-form indicator is not required to show regular type. (See pages 27 - 31 for the various print type forms of the letters of the English, German, Greek, Hebrew, and Russian alphabets.)

Type-Form Indicators for Letters, Numerals, and Compound Expressions

Boldface Type Indicator	⋮• ⋮• ⋮•
Italic Type Indicator	⋮• ⋮• ⋮•
Sanserif Type Indicator	⋮• ⋮• ⋮• ⋮• ⋮• ⋮•
Script Type Indicator	⋮• ⋮• ⋮•

§41. Type-Form Indicators With Letters:

a. The appropriate type-form indicator must be used when it is necessary to show that a letter from any alphabet is printed in a type form other than regular type. A type-form indicator *must always* be followed by an alphabetic indicator.

(1) A

⋮• ⋮• ⋮• ⋮•
⋮• ⋮• ⋮• ⋮•

(boldface English capitalized a)

(2) a

⋮• ⋮• ⋮•
⋮• ⋮• ⋮•

(boldface English uncapitalized a)

(3) α

⋮• ⋮• ⋮•
⋮• ⋮• ⋮•

(boldface Greek uncapitalized alpha)

(4) α

⋮• ⋮• ⋮•
⋮• ⋮• ⋮•

(boldface German uncapitalized ah)

(5) a

⋮• ⋮• ⋮• ⋮•
⋮• ⋮• ⋮• ⋮•

(boldface Russian uncapitalized ah)

(6) A

⋮• ⋮• ⋮• ⋮•
⋮• ⋮• ⋮• ⋮•

(italic English capitalized a)

(7) *a*



(italic English uncapitalized a)

(8) *a*



(italic German uncapitalized ah)

(9) *Œ*



(script English capitalized a)

(10) *α*



(script English uncapitalized a)

(11) *℥*



(script Hebrew aleph)

(12) **H**



(sanserif English capitalized h)

(13) **h**



(sanserif English uncapitalized h)

b. The effect of a type-form indicator extends only to the letter which immediately follows it. Thus, in a sequence of unspaced letters, a type-form indicator must be used before each letter that is not in regular type.

(1) **AB**



(boldface English capitalized a and b)

(2) **ab**



(boldface English uncapitalized a and b)

(3) **α β**



(boldface Greek uncapitalized alpha and beta)

(4) *ab*



(italic English uncapitalized a and b)

(5) *Œ* *ß*



(script English capitalized a and b)

(6) *AbCd*



(italic English capitalized a, boldface uncapitalized b, italic capitalized c, script uncapitalized d)

(7) *pqrs*



(regular English uncapitalized p, boldface q, boldface r, regular s)

(8) *xiyj*



(regular English uncapitalized x, boldface i, regular y, boldface j)

(9) *αa*



(regular Greek uncapitalized alpha, boldface English uncapitalized a)

(10) *HH*



(sanserif English capitalized h, regular English capitalized h)

(11) *p + q = r*



(boldface English uncapitalized p, q, r)

§42. Type-Form Indicators With Numerals:

a. The appropriate type-form indicator must be used when it is necessary to show that numerals are printed in a type form other than regular type. The numeric indicator must always be used between a type-form indicator and a numeral or decimal point.

(1) 0



(boldface zero)

(2) 3



(italic 3)

(3) .3



(italic .3)

(4)



(script 4)

(5) + 0



(ordinary plus, boldface zero)

(6) — 0



(ordinary minus, boldface zero)

b. The effect of a type-form indicator with numerals extends until there is a change in type. Thus, when numerals contain digits in more than one type form, the appropriate type-form indicator and the numeric indicator must be used before each change in type. However, when the change is to regular type, only the numeric indicator is used.

(1) 123



(boldface 1, 2, 3)

(2) 456



(boldface 4, italic 5, regular 6)

(3) 4567



(boldface 4 and 5, regular 6 and 7)

(4) 1234



(regular 1 and 2, boldface 3 and 4)

(5) 100 + 200 = 300



(boldface 1, 2, and 3; all zeros in regular type)

§44. Use and Nonuse of Type-Form Indicators in Technical Texts:

a. When a uniform type form is used throughout a print text for letters, numerals, or other mathematical symbols, the type form must be considered regular type, and type-form indicators must not be used. For example, type-form indicators must not be used when the letters of all formulas throughout a text are uniformly printed in italic type. However, the specific type form must be shown when a different type form is introduced to convey a special mathematical distinction, as when the author uses different type forms to distinguish between two meanings of the same letter. A type form which has no mathematical significance, or which is used only to attract the reader's attention, must not be shown in braille.

- (1) \mathbb{R} denotes the set of rational numbers, and \mathbb{R} denotes the set of real numbers.

b. When boldface type is used in print to identify letters as vectors or to denote a zero as the null vector, the boldface type has mathematical significance and must be shown in braille.

- (1) Is there a vector \mathbf{s} such that $\mathbf{r} + \mathbf{s} = \mathbf{t}$?

- (2) In $p\mathbf{v} = \mathbf{0}$, \mathbf{v} is a vector and $\mathbf{0}$ is the null vector.

- $$(3) \quad \mathbf{a} + \mathbf{0} = \mathbf{a}$$

- (4) $\mathbf{B} = 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

The figure consists of 10 diagrams arranged in two rows of five. Each diagram shows a grid of dots, with some dots being black and others white. The pattern of black dots evolves from left to right. The first diagram shows a small cluster of 5 black dots. The second diagram shows a cluster of 7 black dots. The third diagram shows a cluster of 9 black dots. The fourth diagram shows a cluster of 11 black dots. The fifth diagram shows a cluster of 13 black dots. The sixth diagram shows a cluster of 15 black dots. The seventh diagram shows a cluster of 17 black dots. The eighth diagram shows a cluster of 19 black dots. The ninth diagram shows a cluster of 21 black dots. The tenth diagram shows a cluster of 23 black dots.

Type-Form Indicators for Labeled Mathematical Statements, Words, and Phrases

Opening Boldface Type Indicator

Closing Boldface Type Indicator

Opening Italic Type Indicator

Closing Italic Type Indicator

• • • • •

§45. Type-Form Indicators With Labeled Mathematical Statements:

a. When a labeled mathematical statement such as a theorem, definition, axiom, lemma, etc. is printed in nonregular type, the label must be transcribed as though it were entirely capitalized, and the statement must be preceded and followed by its appropriate opening and closing type-form indicators. The type-form indicators must be preceded and followed by a space.

- (1) **Theorem 4.** *The diagonals of a rectangle are equal.*

- (2) *Definition.* $x + yi = a + bi$, if and only if $x = a$ and $y = b$.

b. In the body of a labeled mathematical statement, if a word or a phrase is printed in a different nonregular type for emphasis or special distinction, the word or phrase must be preceded and followed by its appropriate type-form indicators. When two type-form indicators must be used consecutively, they must be unspaced from each other.

- (1) **Definition.** A *hexagon* is a polygon which has 6 sides.

- (2) *Definition.* **Equal arcs** are arcs that can be made to coincide.

- (3) **Definition.** A polygon that has 5 sides is a **pentagon**.

The image displays a 3x10 grid of dot patterns representing the digits 0 through 9. The patterns are arranged in three rows and ten columns. The first row shows the standard dot patterns. The second row shows the same patterns with a 3D effect, where the top dots are larger and more prominent. The third row shows the patterns with a shadow effect, where the dots are slightly offset to the right and bottom, creating a shadow-like appearance. The background is a light beige color.

§46. Type-Form Indicators With Unlabeled Statements:

a. In an unlabeled statement, a word or a phrase printed in boldface type for emphasis or special distinction must be preceded and followed by the boldface type-form indicators in accordance with the rules of §45 above.

- (1) Name the set of letters in **apple**.

- (2) LCM stands for least common multiple.

b. In an unlabeled statement, an italicized phrase showing emphasis or special distinction and beginning or ending with a numeral, letter, or other mathematical symbol or expression must be preceded and followed by the italic type-form indicators in accordance with §45 above.

However, an italicized word or an italicized phrase both beginning and ending with a word must be transcribed according to the rules of English braille.

- (1) O is the vertex.

- (2) *The common internal tangent is PC.*

The figure displays 20 dot patterns arranged in two rows of ten. Each pattern is a 3x3 grid of dots. The patterns are as follows:

- Row 1:
 - Pattern 1: All dots present.
 - Pattern 2: Top-right dot missing.
 - Pattern 3: Middle-right dot missing.
 - Pattern 4: Bottom-right dot missing.
 - Pattern 5: Top-left dot missing.
 - Pattern 6: Middle-left dot missing.
 - Pattern 7: Bottom-left dot missing.
 - Pattern 8: Top-middle dot missing.
 - Pattern 9: Middle-middle dot missing.
 - Pattern 10: Bottom-middle dot missing.
- Row 2:
 - Pattern 11: All dots present.
 - Pattern 12: Top-left dot missing.
 - Pattern 13: Middle-left dot missing.
 - Pattern 14: Bottom-left dot missing.
 - Pattern 15: Top-right dot missing.
 - Pattern 16: Middle-right dot missing.
 - Pattern 17: Bottom-right dot missing.
 - Pattern 18: Top-middle dot missing.
 - Pattern 19: Middle-middle dot missing.
 - Pattern 20: Bottom-middle dot missing.

- (3) LCD means *least common denominator*.

Figure 1 displays a 2x10 grid of dot patterns representing the digits 0-9. The top row shows digits 0-9, and the bottom row shows digits 10-19. Each digit is represented by a unique arrangement of black dots on a white background.

- (4) *If $a + b = b + a$, then addition is commutative.*

Figure 1 displays a sequence of 20 small plots arranged in two rows of ten, illustrating the evolution of a point cloud over time. The top row shows the initial state with 10 distinct clusters of points. The bottom row shows the state after 100 iterations, where the clusters have merged into a single, more compact shape.

FORMAT (CONTINUED)

§47. Margins for Labeled Mathematical Statements and Formal Proofs:

a. A labeled mathematical statement or a formal proof, such as a theorem, definition, axiom, proposition, postulate, lemma, etc., should be transcribed in the following format:

i. A line must be left blank before the beginning and after the end of the entire labeled statement or formal proof. However, a line should not be left blank before or after a new page-separation line or at the beginning or end of a braille page.

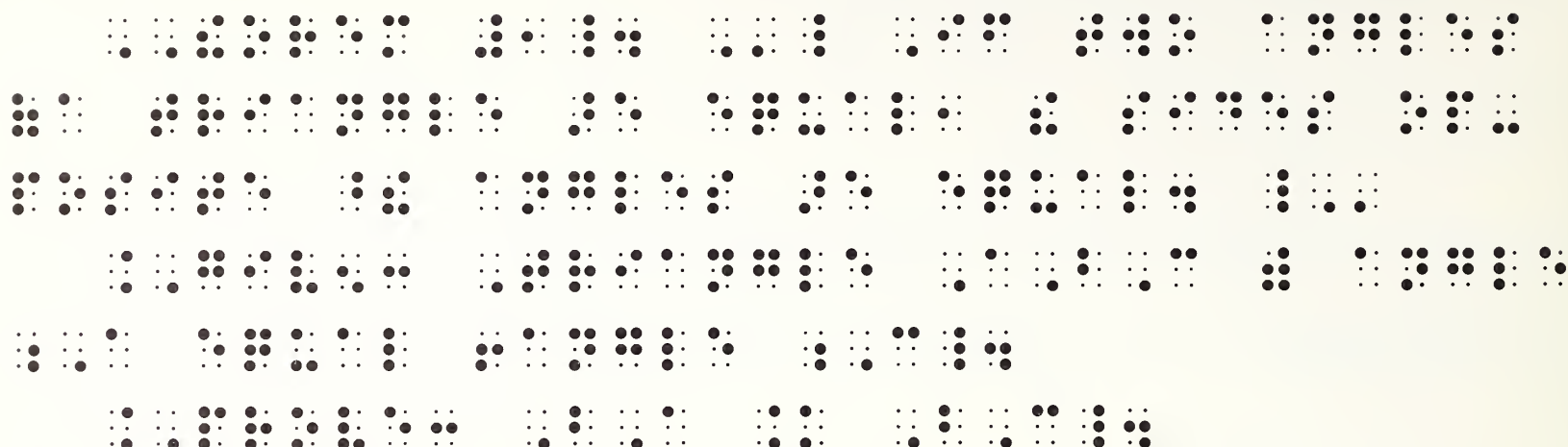
ii. The labeled statement or formal proof must begin in cell 3, and its runovers must begin in cell 1.

iii. If the labeled statement or formal proof contains subheadings such as *Given*, *Prove*, or *Conclusion*, the subheadings must begin in cell 3, and their runovers must begin in cell 1. A line must not be skipped above a subheading. A subheading should be capitalized or italicized in accordance with the print text. If the subheading is in boldface type, it should be entirely capitalized in braille.

(1) **Theorem 1.** If two angles of a triangle are equal, the sides opposite these angles are equal.

Given: Triangle ABC with angle A equal to angle C.

Prove: $BA = BC$.



b. When a formal proof is presented in step-number form and divided into two columns, usually headed *Statements* and *Reasons*, the following format must be used:

i. A line should be left blank before the beginning and after the end of the step-numbered items.

ii. The print columnar form must not be followed in braille. Each item from the *Reason* column must be placed beneath its matching item from the *Statement* column.

iii. The letters "S" for *Statement* and "R" for *Reason* must be placed immediately after the appropriate step number. Similarly, other column headings should be indicated by appropriate letters.

iv. Each step number must begin in cell 1, and any runovers must begin in cell 3.

v. A transcriber's note must be included to explain the change in format and to specify the meaning of the letters used to replace the headings. The note must be placed at the beginning of each volume in which a change of format occurs.

(1) **Theorem 2.** All right angles are equal.

Given: Angle ABC and angle DEF are right angles.

Prove: Angle ABC equals angle DEF.

Proof:

<i>Statements</i>	<i>Reasons</i>
1. Angle ABC and angle DEF are right angles.	1. Given.
2. Angle ABC equals 90 degrees; angle DEF equals 90 degrees.	2. A right angle contains 90 degrees.
3. Angle ABC equals angle DEF.	3. Transitivity postulate.

HOMEWORK

Prepare the following homework for submission to your teacher. If a transcriber's note is required, it should be placed on a separate transcriber's note page in accordance with the rules of the *Code of Braille Textbook Formats and Techniques*. Proofread carefully.

EXERCISE 4

Write these German letters as indicated.

1. Boldface: **a, b, m, n**. **U, B, M, N**. a **U**.
2. Script: *a, b, m, n*. *U, B, M, N*. a *U*.

Write these Greek letters as indicated.

3. Boldface: **α, β, γ, ε**. **Α, Β, Γ, Ε**. a **Α**.
4. Script: *α, β, γ, ε*. *Α, Β, Γ, Ε*. a *Α*.

Write these Hebrew letters as indicated.

5. Script: *ל, נ, ז, ט, צ*. *ל נ*.

Write these Russian letters as indicated.

6. Boldface: **а, б, л, м**. **А, Б, Л, М**. a **А**.
7. Script: *а, б, л, м*. *А, Б, Л, М*. a *А*.

Write these English letters as indicated.

8. Boldface: **a, b, r, s**. **A, B, R, S**. a**A**.
9. Italic: *a, b, r, s*. *A, B, R, S*. a*A*.
10. Script: *a, b, r, s*. *Α, Β, R, S*. a*Α*.
11. Sanserif: **b, c, h, m**. **B, C, H, M**. b**B**.
12. In $\mathcal{R} \times \mathcal{R}$ the two perpendicular vectors **w** and **t** whose sum is **v** are perpendicular components of **v**.
13. Find the angle between the vectors $\mathbf{A} = -3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ and $\mathbf{B} = -6\mathbf{i} - 10\mathbf{j} + 2\mathbf{k}$.
14. Prove that $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{A}$.

Write these numerals as indicated.

15. Boldface: **0, 1, 2, 3, 456, 789**.
16. Italic: *0, 1, 2, 3, 456, 789*.

17. 893,741

18. 999,999

19. 522,489

20. $300 + 300 = 600$

21. Is it possible to generate a **10,000-volt** potential difference by rubbing a comb with wool?
22. A **36-in.** ruler is the same as a *2-yd.* ruler.
23. The filament resistance is lower in a **500-watt** light bulb than in a **100-amp** bulb.
24. A **1000-gram** weight equals a **5.2-oz.** weight.

Theorem 4. If two sides of a triangle are equal, the angles opposite these sides are equal.

Given: Triangle ABC with $BA = BC$.

Prove: Angle A equals angle C.

Proof:

<i>Statements</i>	<i>Reasons</i>
1. Draw BD bisecting angle ABC and meeting AC at D.	1. Every angle has one and only one bisector.
2. Angle 1 equals angle 2.	2. A bisector divides an angle into two equal parts.
3. $BA = BC$.	3. Given.
4. $BD = BD$.	4. Identity postulate.
5. Triangle ABC is congruent to triangle CBD.	5. s.a.s. = s.a.s.
6. Angle A equals angle C.	6. Corresponding angles of congruent triangles are equal.

LESSON 5

SIGNS OF GROUPING

§48. **Use of Signs of Grouping:** The signs of grouping listed below must be used throughout a technical text both for literary and for mathematical material. The parentheses, brackets, and braces of English braille must only be used to enclose literary material on title pages. Any sign of grouping not listed below must be devised using two or more braille symbols whose last cell is ⠠ for the opening sign and ⠡ for the closing sign. A transcriber's note must be included to explain the devised sign of grouping.

While signs of grouping most commonly occur in pairs, the text must be followed when only the opening or only the closing grouping sign is shown in print. No space should be left between an opening or a closing sign of grouping and the material which it encloses.

Parentheses

Opening	(⠠
Closing)	⠡

Brackets (square)

Opening	[⠠ ⠠
Closing]	⠡ ⠡
Boldface Opening	[⠠ ⠠ ⠠
Boldface Closing]	⠡ ⠡ ⠡

Barred Brackets

Opening	[⠠ ⠠ ⠠
Closing]	⠡ ⠡ ⠡

Angle Brackets

Opening	<	⠠ ⠠ ⠠
Closing	>	⠡ ⠡ ⠡

Braces (curly brackets)

Opening	{	⠠ ⠠
Closing	}	⠡ ⠡

Barred Braces

Opening

[

⠠⠠⠠

Closing

]

⠠⠠⠠

Half Brackets

Upper Left

[or ⠠

⠠⠠⠠

Upper Right

] or ⠡

⠠⠠⠠

Lower Left

[or ⠠

⠠⠠⠠

Lower Right

] or ⠡

⠠⠠⠠

Vertical Bars

Single

|

⠠

Double

||

⠠⠠

Boldface Single

|

⠠

Boldface Double

||

⠠⠠⠠⠠

Transcriber's Grouping Symbols

Opening

(no print equivalent)

⠠⠠

Closing

(no print equivalent)

⠠⠠

§49. **Punctuation With Signs of Grouping:** A sign of grouping must be punctuated mathematically and must not itself be considered a punctuation mark. Thus, except for the mathematical comma, hyphen, and dash, the punctuation indicator must be used before a punctuation mark following a sign of grouping.

(1) $(\alpha), (\beta).$

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(2) ("two").

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(3) ("2")

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

(4) (—"two")

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- (4) (were you sure of your answer)

- (5) (*his answer is wrong*)

- (6) (“in all cases it holds true”)

The figure consists of 12 diagrams arranged in a single row, each showing a 4x4 grid of dots. The dots represent particles in a lattice. The sequence shows the growth of the lattice over time. The first diagram has a single dot at (1,1). The second has dots at (1,1) and (1,2). The third has dots at (1,1), (1,2), and (1,3). The fourth has dots at (1,1), (1,2), (1,3), and (1,4). The fifth has dots at (1,1), (1,2), (1,3), (1,4), and (2,1). The sixth has dots at (1,1), (1,2), (1,3), (1,4), (2,1), and (2,2). The seventh has dots at (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), and (2,3). The eighth has dots at (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), and (2,4). The ninth has dots at (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), and (3,1). The tenth has dots at (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), and (3,2). The eleventh has dots at (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), and (3,3). The twelfth has dots at (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), and (3,4).

- (7) (This cannot be.)

- (8) (into every dividend)

- (9) (this fact must be looked into)

- (10) (“By your leave.”)

- (11) (to within a tolerance of 5 cm)

- (12) (by your calculations)

iii. The whole- or part-word contractions for *and*, *for*, *of*, *the*, *with*.

- (1) Mary (and Sandra) can add.

- (2) (*and*, in addition)

- (3) {Andrea, Andrew, Sandra}

- (4) We are (for and with) you.

- (5) (For example.)

-

-

-

-

-
- The figure shows a 3x10 grid of dot patterns. Each cell contains a 3x3 grid of dots. The patterns are as follows:
- | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

-

-

-

-

- [illegible]

-

-

- (6) (1-to-1 correspondence)

- (7) $(.1 < .2)$

- (8) $(-10 \text{ and } +4 \text{ is } -6)$

- (9) (2 is imaginary; ['2' is real])

- (10) (2 and 3)

§52. Signs of Grouping With Letters:

- a. The English letter indicator must not be used when a single English letter or an unspaced combination of letters in regular type is entirely enclosed within signs of grouping.

- (1) (a), (b), and (c).

- (2) $[x]$, $[x]$, $\{R\}$, $|y|$, $\|f\|$

- (3) (ab) and (cd) are not equal.

- (4) (xy) and (XY)

- b. When a single English letter or an unspaced combination of letters is in direct contact with only its opening or closing sign of grouping, the English letter indicator must be used or must not be used as though the grouping signs were not present. However, the English letter indicator must not be used when a grouping sign carries a prime or other modifying symbol.

- (1) (a, b, and c)

- (2) (p is less than q)

- (3) ("M is greater than N")

(4) (x's and y's)

(5) (nth root of x)

(6) (x-intercept)

(7) (not-p)

(8) (ab is the same as ba)

(9) (side AB)

(10) $(-r, +s, \text{ and } -t)$

[illegible]

(11) (b' is read "b prime")

(12) $(a = b, b = c, ab = cd)$

(13) $(j = 1, 2, 3, \dots, n)$

The figure consists of seven 5x5 dot grids arranged horizontally. Each grid contains a pattern of black dots. The pattern starts in the first grid as a small cluster of dots and grows progressively in each subsequent grid, eventually filling most of the 5x5 grid by the seventh stage.

(14) a) $3 \times 4 = ?$

The sequence of dot patterns is as follows:

- Pattern 1: Top row has 1 black dot (center); bottom row has 2 black dots (left and center).
- Pattern 2: Top row has 2 black dots (center and right); bottom row has 2 black dots (left and center).
- Pattern 3: Top row has 2 black dots (center and right); bottom row has 3 black dots (left, center, and right).
- Pattern 4: Top row has 3 black dots (center, right, and bottom-right); bottom row has 3 black dots (left, center, and right).
- Pattern 5: Top row has 3 black dots (center, right, and bottom-right); bottom row has 4 black dots (left, center, right, and bottom-right).
- Pattern 6: Top row has 4 black dots (center, right, bottom-right, and bottom-center); bottom row has 4 black dots (left, center, right, and bottom-right).
- Pattern 7: Top row has 4 black dots (center, right, bottom-right, and bottom-center); bottom row has 5 black dots (all dots in the row).
- Pattern 8: Top row has 5 black dots (all dots in the row); bottom row has 5 black dots (all dots in the row).
- Pattern 9: Top row has 5 black dots (all dots in the row); bottom row has 6 black dots (all dots in the row and the center dot of the top row).
- Pattern 10: Top row has 6 black dots (all dots in the row and the center dot of the top row); bottom row has 6 black dots (all dots in the row and the center dot of the top row).

(15) $t]'$ and $v]'$ have unique meaning.

c. The English letter indicator must be used with any English letter printed in nonregular type even though it is enclosed within, or in contact with, signs of grouping.

(1) (j)

● ● ● ● ●

(2) **(r, s, and T)**

d. The appropriate alphabetic indicator must be used with any letter from the German, Greek, Hebrew, or Russian alphabets even when enclosed within, or in contact with, signs of grouping.

- (1) (a)



- (2) (α)



- (3) (\mathbb{N} and \mathbb{A})



§53. Signs of Grouping With Abbreviations: When an abbreviation occurs within grouping signs, no space should be left between the abbreviation and its related sign of grouping. The English letter indicator must be treated as though the grouping signs were not present.

- (1) 1 liter (l)



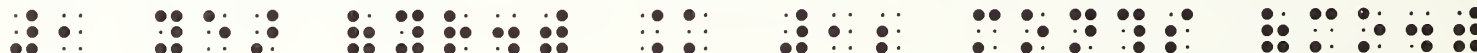
- (2) 1 liter (l.)



- (3) 1 year (yr) = 12 months (mo)



- (4) 1 year (yr.) = 12 months (mo.)



- (5) (N 30 degrees W)



- (6) (N. 30 degrees W.)



- (7) (1 light-yr)



- (8) (1 light-yr.)



- (9) (g-mole)



- (10) (d-c means “direct current”)



(4) $(\text{rate}) \times (\text{time}) = (\text{distance})$



(5) $(2 \times 3)\$ = 6\$$



(6) Do examples (1)-(10).



(7) $(1) \dots (10)$



(8) $(2x - 3y) \text{ mi.}$



(9) Solve for x ($x > 0$).



(10) $f(x) = |x|$



(11) $P(x, y) = \phi(\theta)$



(12) $A(2) + B(1) + C(2) + D = 0$



(13) $3(x - 1)(x + 4) < 0$



(14) $[-4 - (-1)] + [-1 - (-3)]$



(15) $\{(1, 0), (1, 1)\}$



c. A space must be left between an opening and a closing sign of grouping when the blank space between them in print does not represent an omission.

(1) Parentheses () are symbols of grouping.



§57. Transcriber's Notes:

a. The transcriber's grouping symbols $\begin{smallmatrix} \cdot\cdot & \cdot\cdot \\ \bullet & \bullet \end{smallmatrix}$ must be used to enclose a transcriber's note which has been inserted in the text. They must not be used to enclose transcriber's notes on preliminary pages at the beginning of a braille volume.

b. A transcriber's note of seven words or less may be inserted directly into the text where it applies, leaving one space before and after the note. Longer notes must be inserted at a convenient point nearest the material referred to and must be placed, indented, and run over in accordance with the rules of the *Code of Braille Textbook Formats and Techniques*.

c. The same rules governing the punctuation and contraction of expressions with signs of grouping also apply to transcriber's grouping symbols. The numeric indicator must be used after the opening transcriber's grouping symbol, and the English letter indicator must be used or must not be used as though the grouping signs were absent.

- (1) What place value does the red 2 have in 22?

- (2) In $4 + 5$, 4 is an even number.

- (3) An **F** denotes negation.

- (4) Which number is greater **2** or 3?

NUMERALS IN NONDECIMAL BASES

§58. Representation of Numerals in Nondecimal Bases:

a. A nondecimal numeration system may use letters to supplement the 10 Arabic numerals and to represent the additional digits required. In such instances, uncapitalized letters must be used in braille. If capitalized letters are used in print, a transcriber's note must be included to explain the change to uncapitalized letters.

The letters used to represent digits in a nondecimal numeration system must be treated as numerals, and the numeric indicator must be used where required. Such numerals are mathematical symbols and must be punctuated accordingly.

- (1) 13T8 and T1E5 are base 12 numerals. T represents 10 and E represents 11.

13T8 T1E5 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

- (2) In base 12, can —t2e be a negative number?

—t2e 10 2 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

- (3) The digit "T" denotes "ten."

T 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

- (4) (T) denotes "ten" in base 12.

(T) 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

- (5) Add in base 12: 24T, 3E, 5E.

24T 3E 5E 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

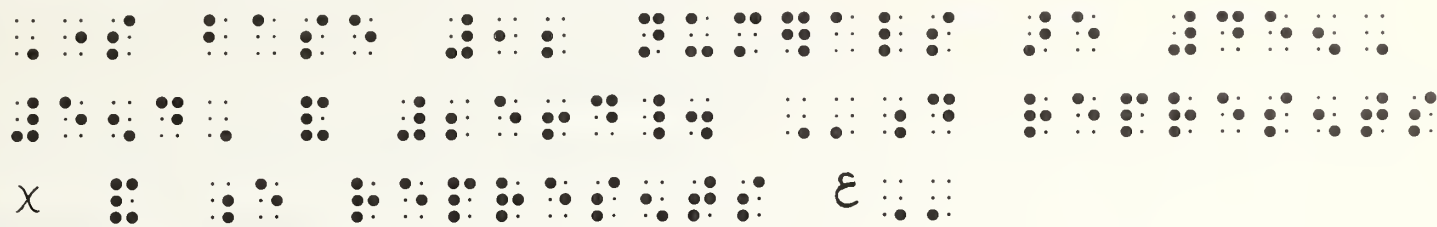
- (6) The letters T and E represent ten and eleven in TE5.

TE5 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

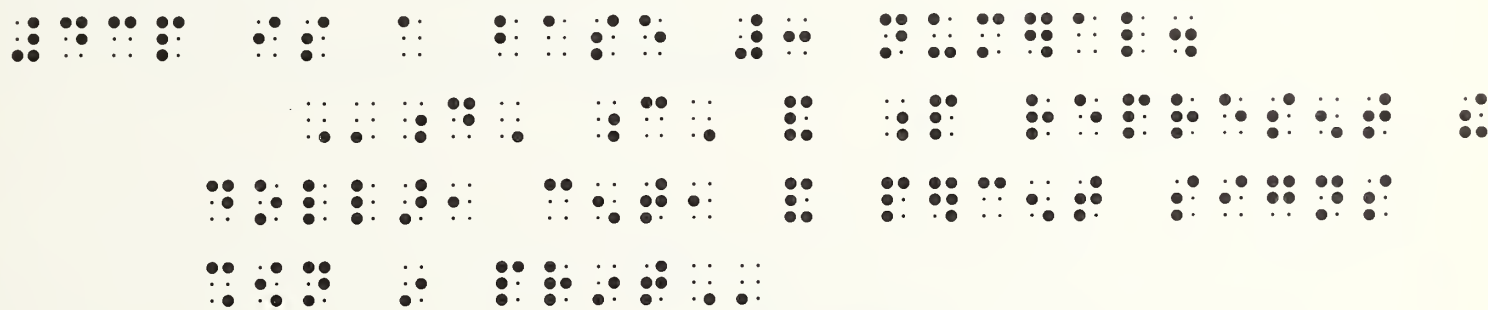
b. A nondecimal numeration system may use special signs, such as χ (dek) and \mathcal{E} (el), in conjunction with the 10 Arabic digits; or it may use an arbitrary set of signs, such as \$ ϕ %, which contains no Arabic digits. In such cases, one-cell symbols preferably chosen from the letters of the English alphabet must be selected to represent these special signs. A transcriber's note must be inserted to specify the meanings assigned to these symbols, and must also include a drawing of any print sign lacking an equivalent symbol in the code.

The symbols for these special signs in nondecimal numeration systems must be treated as numerals, and the numeric indicator must be used where required. Such signs are mathematical symbols and must be punctuated accordingly.

(1) Some base 12 numerals are $\chi \mathcal{E} 5$, $\mathcal{E} 5 \chi$, and $2 \mathcal{E} 6 \chi$.



(2) \$ ϕ % is a base 3 numeral.



HOMEWORK

Prepare the following homework for submission to your teacher. If a transcriber's note is required, be sure to include it in its proper place. Proofread carefully.

EXERCISE 5

1. ("20%"), ("25%"), and ("50%") are the same as what fractions?
2. Find the volume of a sphere having a radius of 6 inches. (Use $\pi = 3.14$)
3. Draw a regular hexagon (a figure with 6 equal sides and 6 equal angles) inside a circle.
4. What is the least common denominator (L.C.D.) of 8 and 16?
5. Define a geometric progression (GP).
6. Prove: In logic, the result of each value of x (from an appropriate domain) behaves as a "statement function."
7. Multiplication (not addition) is the inverse of division.
8. For every sequence (in exercise 19) determine the difference.
9. Illustrate the way in which each member of X is a member of some (and, hence exactly one) member of A .
10. Is {Andrew, Albert, Andrea} a true set?

11. Use numerals to prove $a + b = b + a$ (for example, if $a = 2$ and $b = 3$, then $2 + 3 = 3 + 2$).
 12. Complete: A (formal) proof is a finite column of formulas
 13. Find the sum (of the first seven terms) of the series 12, 9, 6,
 14. How do we derive a (proof) schema of a (theorem) schema?
 15. Write a numeral for the cardinal number of {the set of eggs in a dozen}.
 16. Can you find the answer (without using pencil and paper) to problem 4?
 17. Solve examples (1), (2), and (3). Is (.5) and (.6) an accurate estimate?
 18. A conical pile of sand 6 ft. high has a base of 27 sq. ft. How many loads of sand are in the pile? (1 load = 1 cu. yd.)
 19. Estimate the sizes of angles (d) and (e) using the measurements of the angles in (A), (B), and (C). Check, using (ab and de).
 20. If $x > y$ (x is greater than y) and $x = 10$, what is the relationship when $y = 4$?
 21. For vectors (a, b, c) and real numbers $(p$ and $q)$ can we say that $a + (b + c) = (a + b) + c$?
 22. (γ, θ, ϕ) denotes what kind of coordinates?
 23. What meaning does $u]'$ and $w]'$ have?
 24. State the axiom for the inequality $a + c < b + c$, ($a < b$).
 25. 1 liter (l) is equal to how many cubic centimeters (cc)?
 26. How many days are there in 1 year (yr)?
 27. Write a description of the following set: {Pa., O., Ky., Va., Md.}.
 28. In how many ways is it possible to set up a 1-to-1 correspondence between $\{a, b, c, d\}$ and $\{w, x, y, z\}$?
 29. Can we say (a, b, c, \dots) is infinite?
 30. (100, 200, _____, 400)
 31. Picture the set of points on a real number line that are in the following intervals: $\langle -3, 2 \rangle$; $[0, 1 \rangle$; $[2, \pi]$.
 32. Consider $x > 4$. If $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the replacement set for the variable, what is the solution set?
 33. Mark points corresponding to the ordered pairs (4 and 2), (5 and 0) on a set of axes. What does each point represent?
- Graph.
34. $|x + y| = |x| + |y|$
 35. $g(x) = (x - 1)(x - 2)(x - 3)$ and $G(t) = t(t - 1)(t - 2)(t - 3)$
- Simplify.
36. $(y + 6) - (6 - y)$
 37. $10a + [3a - (5a - 4)]$

38. $[(a + b) - 2c] [(a + b) + 3c]$

39. $|3 - 5| \cdot (x - 5)$

Answer the following questions.

40. Show that $M = \{(4, 2) + s(1, -1)\}$.

41. Does $x = 0$ when $\phi(\theta) = P$ is a point on the y-axis?

42. Tell what each of the following signs mean: $(+, -, \times, \div)$ and $(=, <, >)$.

43. Explain why brackets $[]$ and braces $\{ \}$ are called symbols of aggregation.

44. For what value(s) of v will $v - 2$, $2v - 6$, and $4v - 8$ form an A.P.?

45. Which numeral is greater: 3 or 3?

46. What value has the digit t in $e5t7$?

47. Express each base 12 numeral as a decimal numeral: $4TEE$; $T1E5$; $E5T$.

LESSON 6

SUPERSCRIPTS, SUBSCRIPTS, AND LEVEL INDICATORS

A mathematical expression can contain symbols placed above or below the normal baseline of writing. Superscripts appear above the baseline; subscripts appear below the baseline. Since it is not practical to write on different levels in braille, level indicators are used to identify the level of a superscript or subscript in relation to the baseline of writing.

Level Indicators

Superscript	$\overset{\cdot}{\cdot}$
Superscript With Superscript	$\overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$
Superscript With Superscript With Superscript	$\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$
Subscript	$\underset{\cdot}{\cdot}$
Subscript With Subscript	$\underset{\cdot}{\cdot} \underset{\cdot}{\cdot}$
Subscript With Subscript With Subscript	$\underset{\cdot}{\cdot} \underset{\cdot}{\cdot} \underset{\cdot}{\cdot}$

§59. **Superscript Level Indicators:** The superscript level indicator $\overset{\cdot}{\cdot}$ must be used to show that the symbols immediately following it appear on the first level above the baseline of writing. Superscripts may carry superscripts of their own. In such cases, the superscript indicator must be doubled, tripled, etc. to indicate superscripts on the second, third, or higher levels of writing.

(1) x^2

$\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$

(2) π^2

$\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$

(3) n^m

$\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$

(4) 24^7

$\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$

(5) $3^{-0.05T}$

$\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$

(6) $(y - k)^3$

$\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$

(7) $A^{(k+1)+m}$

$\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$

(8) n^{a^b}



(9) $n^a + {}_1b + {}_1c + {}_1$



§60. Subscript Level Indicators:

a. Except as stated in b below, the subscript level indicator dots 2, 3, 4 must be used to show that the symbols immediately following it appear on the first level below the baseline of writing. Subscripts may carry subscripts of their own. In such cases, the subscript level indicator must be doubled, tripled, etc. to indicate subscripts on the second, third, or lower levels of writing.

(1) f_n



(2) \mathfrak{z}_k



(3) $a_{(k+1)}$



(4) $A_n \sqcap {}_1$



(5) 10_7



(6) δ_{ij}



(7) 10_8



(8) $?_3$



(9) $(CO)_2$



(10) n_{xy}



(11) $P_{x+{}_1y+{}_1z+{}_1}$



b. The subscript level indicator must not be used before a *numeric* subscript on the *first* level below the baseline of writing if the numeral is a right-subscript to a *letter*. The numeric subscript may contain a comma, a decimal point, or one or more prime signs. The letter may be taken from any alphabet in any type form and may be part of a 2-letter chemical abbreviation. However, the letter must not be used as a numeral in a nondecimal numeration system or as part of a word or abbreviation.

The subscript level indicator must be used with a numeral on the first level below the baseline if a subscript contains any symbol other than a numeral with its comma or decimal point, or if the subscript carries a superscript or subscript of its own. Subscripts on the second or lower levels of writing always require their appropriate subscript level indicators.

(1) x_1

⠠⠭⠨

(2) ax_2

⠠⠁⠭⠨

(3) A_{56}

⠠⠠⠼⠨⠠⠼⠨

(4) $x_{1,000}$

⠠⠭⠨⠠⠼⠨⠠⠼⠨⠠⠼⠨⠠⠼⠨

(5) $x_{5.3}$

⠠⠭⠨⠠⠼⠨⠠⠼⠨

(6) $x_{.7}$

⠠⠭⠨⠠⠼⠨

(7) α_2

⠠⠠⠼⠨⠠⠼⠨

(8) Σ_0

⠠⠠⠼⠨⠠⠼⠨

(9) i_1

⠠⠠⠼⠨⠠⠼⠨

(10) CO_3

⠠⠠⠼⠨⠠⠼⠨⠠⠼⠨

(11) Na_3

⠠⠠⠼⠨⠠⠼⠨⠠⠼⠨

(12) TE_{12}

⠠⠠⠼⠨⠠⠼⠨⠠⠼⠨⠠⠼⠨

(7) $7_8 - 4_8$

$$(8) \quad \pm a_i b_j c_k d_l$$

(9) five₃ + five₃

$$(10) \quad a_1 a_2 + b_1 b_2 + c_1 c_2$$

$$(11) \quad \mathbf{v} = v_1 \mathbf{i}_1 + v_2 \mathbf{i}_2 + \dots$$

$$(12) \quad a_0 b_k + a_1 b_{k-1} + \dots + a_k b_0$$

b. The baseline indicator must be placed before a sign of grouping when a level indicator is in effect and the grouping symbol appears on the baseline of writing. However, the baseline indicator must not be placed after a numeric subscript not requiring a subscript indicator.

$$(1) \quad (x^2 + y^2) - (x^2 + y^2)$$

$$(2) \quad (a^2)^8(+2b)^3$$

$$(3) \quad \{f_n\}$$

$$(4) \quad |a_m - a_n|$$

(5) $(\mathbf{x}_1\mathbf{y}_1 + \mathbf{x}_2\mathbf{y}_2)$

(6) $([\text{CH}_2]_2 \text{CH})$

$$(7) \quad f_1(x) = g(x) \cdot q_2(x) + f_2(x)$$

c. No space should be left between an abbreviation and a related indicator. When it is necessary to return to the baseline, no space should be left before the baseline indicator.

(1) $144 \text{ ft}^2 + 144 \text{ ft}^2$



(2) $2\text{ m}^3 + 2\text{ m}^3$



(3) $2 \text{ m.}^3 + 2 \text{ m.}^3$



§62. Effect of Level Indicators:

a. The effect of one level indicator is terminated by another level indicator.

(1) $8x^5 - 6x^3y^2$



$$(2) \quad (r_a + s^2)$$



(3) n^{xy}



(4) $a^{(m^n)}$



(5) 10^{-4}



(6) P_{xyz}



b. The effect of a level indicator is terminated by the punctuation indicator. Thus, the punctuation indicator must be used after a word or an abbreviation in a superscript or subscript if the punctuation following it is on the baseline of writing. If the punctuation indicator is required in a superscript or subscript, the appropriate level indicator must be used before the punctuation indicator to show continuation of the level in effect.

(1) Multiply: $2^{10} \times 2^3$.



(2) e_i .



(3) The x^2 's and x^3 's.

(4) y_1 's . . . y_n 's

(5) S_{angle} .

(6) $A^m + m + m's$.

c. The effect of a level indicator is terminated by a comma unless the comma occurs in a long numeral. If a comma comes between two separate items in a superscript or subscript, the symbol ⋅ must be used to replace the comma or the comma and a space.

In such cases, the comma represented by this symbol does not terminate the superscript or subscript level already in effect. The symbol $\overset{\cdot}{,}$ must not be used to replace the comma on the baseline of writing.

(1) x^2, y^2, z^2

(2) (a_1, a_{1i}, a_{2i})

(3) Add: $2_{\text{five}}, 3_{\text{five}}, 4_{\text{five}}$.

The figure shows a 4x10 grid of dots. The dots are arranged in a pattern that suggests a sparse matrix structure, with some rows having more dots than others. The dots are arranged in a pattern that suggests a sparse matrix structure, with some rows having more dots than others.

(4) $e^{1,000}$

(5) $x^{1, 2}$

(6) $x^{1,j}$

(7) $x_{1,2}$

(8) $X_{1, j, k}$

$$(9) \quad X_{n-1, n-1}, X_{n-1, n}, X_{n, n-1}$$

(6) $(-3)^{-2} + ^{+2}$



(7) ${}_3P_2$



(8) ${}_{48}C_9 \times {}_4C_4$



(9) ${}_nP_r = K({}_{n-1}P_{r-1})$



(10) ${}_y^x n$



(11) ${}_x^y n$



b. The appropriate superscript indicator must be repeated before a left-superscript when two superscripts are consecutive, but one applies to the expression preceding it and the other to the expression following it.

(1) $p^b {}^c q$



c. The subscript indicator must be used before a left-subscript when two subscripts are consecutive, but one applies to the expression preceding it and the other to the expression following it.

(1) $P_b {}_c Q$



(2) $P_{12} Q$



d. When a left-superscript is shown with a single-letter abbreviation, the English letter indicator is not used.

(1) $19,872,369 {}^\circ C$



(2) $10^{-23} {}^\circ F$



§64. Simultaneous Superscripts and Subscripts: When a mathematical expression carries a superscript and a subscript printed directly above and below each other, the subscript must be shown first, even if the subscript is numeric and requires no subscript level indicator.

(1) x_a^n



(2) 10_8^3



(3) x_1^2



(4) $\sum_0^n a_k$



(5) $a_1^2 + b_1^2 + c_1^2$



(6) $a_i^2 + b_i^2 + c_i^2 + \dots$



(7) $[t]_0^4$



(8) $s]_{t=a}^{t=b} = b - a$



(9) $n_{a b c}^{x y z}$



(10) ${}_m^n x$



§65. Nonsimultaneous Superscripts and Subscripts: When a mathematical expression carries a superscript and a subscript not printed directly above and below each other, the superscript and subscript must be transcribed in the same order as in print, and the baseline indicator must be inserted between them.

(1) a_m^k



(2) 10_8^3



(3) $2 \times 10_6^2 + 3 \times 10_6^1 + 2$



$$(3) p_1^{\alpha_1} \dots p_r^{\alpha_r}$$



$$(4) A_{h_k+1}^{r_k+1}$$



$$(5) {}^n\mathbf{X}$$



$$(6) {}^a\mathbf{X}$$



c. Superscript With Subscript With Subscript:



$$(1) x^{nr_j}$$



d. Superscript With Subscript With Superscript:



$$(1) x^{yr^n}$$



e. Superscript With Superscript With Subscript:



$$(1) x^{yz^n}$$



f. Subscript With Superscript:



$$(1) x_{yr}$$



$$(2) x_{2^n}$$



$$(3) {}_y{}^n\mathbf{X}$$



$$(4) {}_{n_n}\mathbf{X}$$



g. Subscript With Superscript With Superscript:



(1) $x_{y_{r^n}}$



h. Subscript With Superscript With Subscript:



(1) $x_{y_{r_n}}$



i. Subscript With Subscript With Superscript:



(1) $x_{y_{r_n}}$



FORMAT (CONTINUED)

§69. Margins for Itemized Material With Subdivisions:

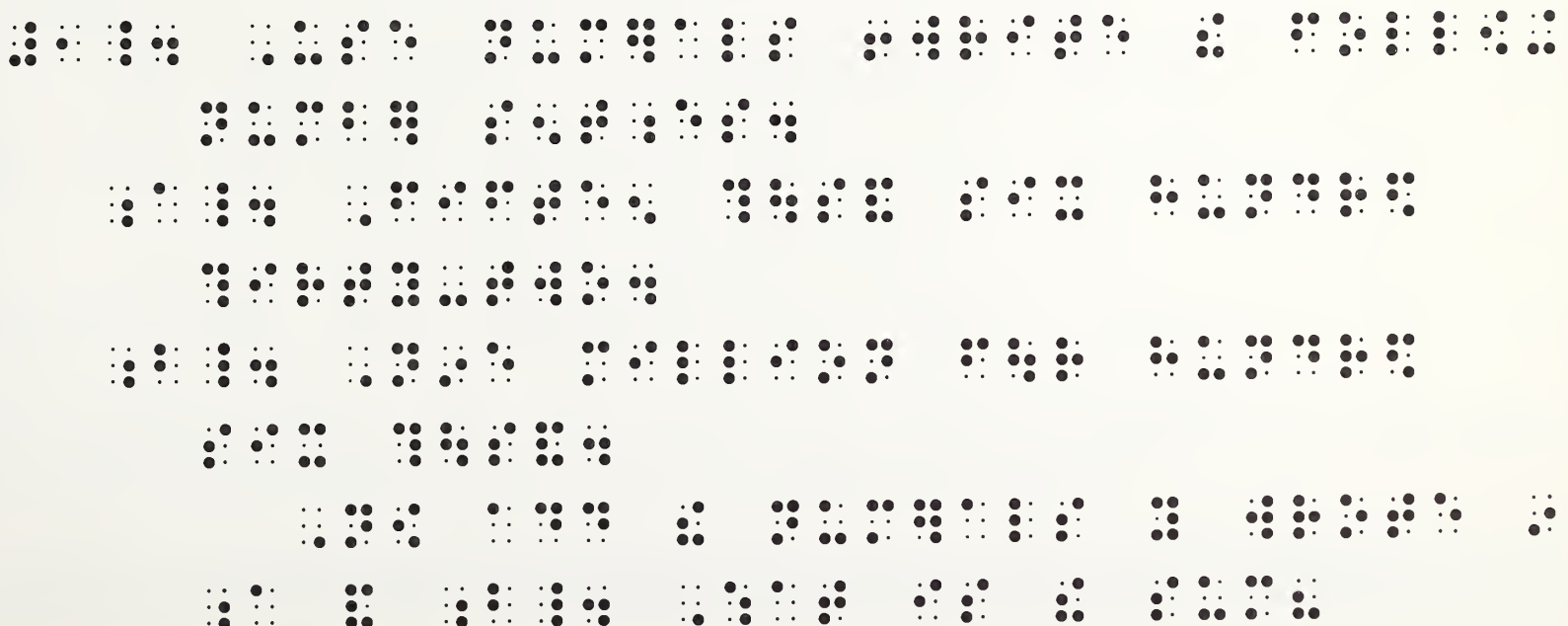
a. In transcribing itemized material with lettered or numbered subdivisions, the main item designation must begin in cell 1, and its runovers must begin in cell 5. Each lettered or numbered subdivision must begin in cell 3, and its runovers in cell 5. If any item has more than one paragraph, each new paragraph must begin in cell 7, and its runovers in cell 5.

(1) 1. Use numerals to write the following number sentences.

a. Fifteen thousand six hundred thirty-two.

b. Nine million four hundred six thousand.

Now add the numerals you wrote in a and b. What is the sum?



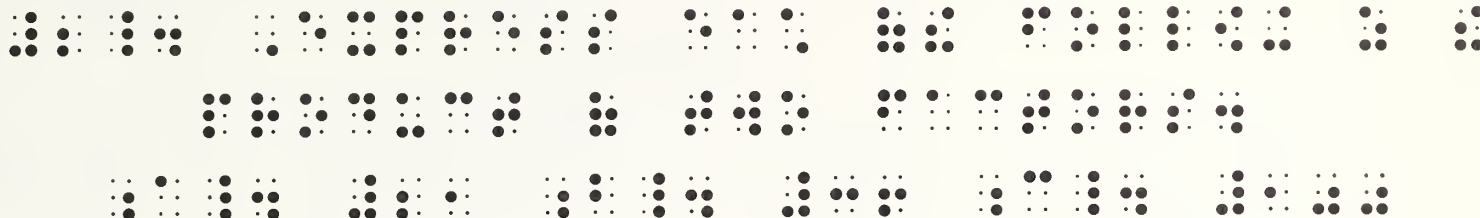
(1) 1. Copy and multiply.

a. 170×71 b. 1.25×12



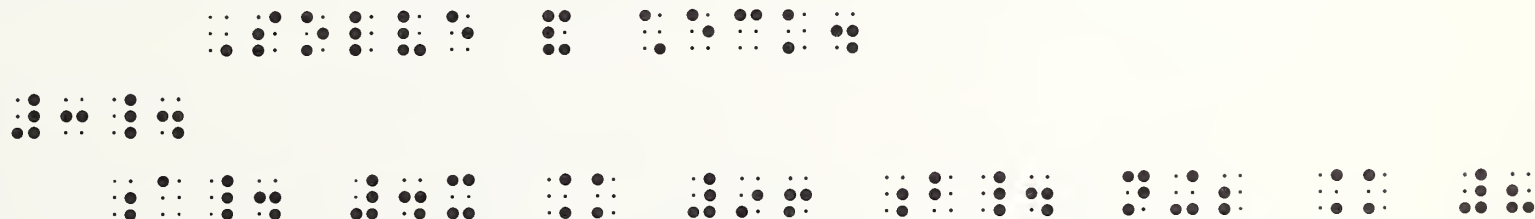
(2) 2. Express each of the following as the product of two factors.

a. 21 b. 36 c. 100



(3) Solve and check.

3. a. $4x = 96$ b. $n + 2 = 8$



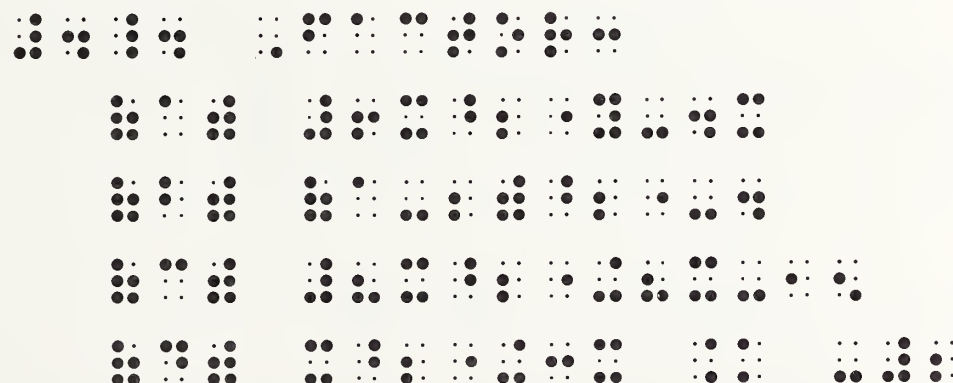
(4) 4. Factor:

(a) $6x^2y - 4x$

(b) $(a - 2)^2 - 4$

(c) $8x^2 + 8x - 15$

(d) $x^2 + 3x = -2$



c. When itemized material is arranged in tabular form so that items are numbered at the margin and subdivisions are aligned beneath lettered column headings, the material should be transcribed in one of the following ways:

i. If all the columns can be accommodated across the braille page, the print columnar arrangement must be followed. Each problem number must begin in cell 1. The letter identifying each column must be aligned with the first cell of the related column. A blank line must be left above and below the lettered column headings. At least two blank cells must be left between columns.

(1) Find the answers mentally.

a	b	c
1. $16 + 9$	$17 + 4$	$14 + 23$
2. $46 + 15$	$87 + 12$	$95 + 54$
3. $157 + 452$	$134 + 63$	$458 + 12$

Braille representation of the above table.

Braille representation of the first row of the table.

Braille representation of the second row of the table.

Braille representation of the third row of the table.

Braille representation of the first row of the table.

Braille representation of the second row of the table.

Braille representation of the third row of the table.

Braille representation of the fourth row of the table.

Braille representation of the first row of the table.

Braille representation of the second row of the table.

Braille representation of the third row of the table.

Braille representation of the fourth row of the table.

Braille representation of the first row of the table.

Braille representation of the second row of the table.

Braille representation of the third row of the table.

Braille representation of the fourth row of the table.

ii. If all the columns cannot be accommodated across the braille page, each subdivision in each problem must be lettered individually, and the format in a above must be followed.

(1) Mixed practice.

a	b
1. $17 + 24 + 13 + 26 = \underline{\quad}$	$72 + 19 + 11 + 18 = \underline{\quad}$
2. $4 \times 17 \times 25 = \underline{\quad}$	$16 \times 8 \times 17 = \underline{\quad}$
3. $8 = 5\% \text{ of } \underline{\quad}$	$120 = 300\% \text{ of } \underline{\quad}$

Braille representation of the first row of the table.

Braille representation of the second row of the table.

Braille representation of the first row of the table.

Braille representation of the second row of the table.

Braille representation of the third row of the table.

Braille representation of the fourth row of the table.

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Braille representation of the seventh row of the table.

Braille representation of the eighth row of the table.

Braille representation of the first row of the table.

Braille representation of the second row of the table.

Braille representation of the third row of the table.

Braille representation of the fourth row of the table.

Braille representation of the fifth row of the table.

Braille representation of the sixth row of the table.

Braille representation of the seventh row of the table.

HOMEWORK

Prepare the following homework for submission to your teacher. Proofread carefully.

EXERCISE 6

- Tell what number each of the following names.
a. 2^2 b. 10^3 c. $(3.15)^4$
- Which term of $(a + b)^{12}$ has the factor b^5 ?
- Find the r th term of $(x + y)^n$.
- What number is named by each of the following: 6^2 , 5^3 , and 7^4 ?
- Use table 5 to approximate $e^{0.27}$.
- Show that these number sentences are true.
a. $4^2 \times 4^3 = 4^5$ b. $15^3 \times 15^4 = 15^7$
c. $e^{1,000} \times e^{2,000} = e^{3,000}$
- Factor:
(a) $4x^4 + 19x^2y^2 + 49y^2$ (b) $100a^2b^2c^2 - 1$
(c) $x^{2a+2} + x^{2a+1} + x^{2a}$
- Simplify: $10^{0.30} \div 10^{-0.66}$; $(x^3 - y^3)^2 - (x^3 + y^3)^2$.
- $12 \text{ ft}^2 \times 12 \text{ ft}^2 = ? \text{ ft}^2$
- $14 \text{ m}^3 + 6 \text{ m}^3 = ? \text{ m}^3$
- $21 \text{ m}^3 + 39 \text{ m}^3 = ? \text{ m}^3$
- What is a 3^2 -dimensional system?
- Prove that $f^{(n)}(x)$ exists.
- Substitute to find $A^n + n's$.
- Use a computer to find 9^{99} .
- Find the limit as x approaches 0.6 of $2^{25x^2} - 10x - 1$.
- When will $E[x^2] = e^{2(a+b^2)}$?
- Name the numeral in base ten equal to:
a. 47_8 b. 34_6 c. 1101_2
- Add in base 12: $27TE_{12}$ and $E5T_{12}$.
- If the sets V_k expand as k increases, what happens to the sets C_k ?
- Use 6_8 , 5_8 , 13_8 to write three examples in base 8.

22. If $u_1 = x_1 - 68$, $x_1 - 68$ yields what values of u_1 's?
23. Add: 3_{five} , 4_{five} , and 2_{five} .
24. Simplify: $x_a + bx_a + cx_a$.
25. Show that $\{a_{2n}\}$ and $\{a_{2n+1}\}$ are monotonic.
26. Use the formula on page 372 to find $a_{\overline{32}}$ and $s_{\overline{32}}$ at 2%.
27. What do we know if $P_{n_r} = (x_{n_k}, y_{n_k})$?
28. Generalize Eq. 5 for the case of k samples of size n_1, n_2, \dots, n_k .
29. Use $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$ to denote the direction vectors v_1 and v_2 .
30. Use Table 4 to find the values of $x_{.05}$ and $x_{1.000}$.
31. Prove that if $a_1b_2 - a_2b_1 = 0$, then the graphs of the equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ are either parallel or coincident lines.
32. Choose x_1 's from $(x_1t + x_2)^5$.
33. The notation $m_{k, a}$ means "the k th moment about the point a ."
34. Find $m_{1, 0}$, $m_{2, 0}$, and $m_{3, 0}$ for the given functions.
35. Graph: $\|f - F\|_{[a, b]} < \epsilon$.
36. $m_{(k_1, a_1)(k_2, a_2)}$ is defined in Theorem 4.
37. Evaluate $F_{.10; 1}$.
38. Find the number of $n_{\text{reg. triangles}}$ in the problem.
39. Find $D[h]_{h=6}$ in the given problems.
40. $E(x) = ae^{a(t-1)}]_{t=1} = a$ occurs in the Poisson distribution.
41. Explain the meaning of $x^2 + 4 + 6 + \dots + n$.
42. $8^6 + \text{---} = 8^{10}$.
43. What do the three dots mean in $R_{s_1, s_2, \dots}$?
44. Substitute and use $a^1b^2c^3 \dots m^n$.
45. Do many of the sets V_n correspond to the sets $U_{n_1} \dots U_{n_k}$?
46. If P_n denotes the n th prime, show that $P_1P_2 \dots P_n + 1$ is not a perfect square.
47. Which is larger $e^{3.14159 \ 26535}$ or π^e ?
48. Find the sums:

a. $+1.9 + +12.6$	c. $-7.12 + -8.13$
b. $+10.3 + -20.6$	d. $ -2 + -3 $
49. What is $(-4)^{-6} + ^66$?
50. The permutations of the letters a, b , and c taken two at a time are ab, ac, ba, bc, ca, cb . Thus, ${}_3P_2 = 6$.
51. If ${}_nP_5 = 42{}_nP_3$, find n .

52. ${}_nC_r + {}_nC_{r-1} = \underline{\quad ? \quad}$
53. Write these notations:
 (a) $x^a y$ (b) $X_a{}_b Y$ (c) $X_{1\ 2} Y$
54. P_k^n is a notation used for permutations.
55. $3 \times 10_6^2 + 4 \times 10_6^1 + 7 = \underline{\quad ? \quad}$
56. When will $r_{p_1}^2 = .25\pi^2$?
57. Use the summation sign to express $x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2$.
58. $4 \times 10_8^2 + 2 \times 10_8^1 + 5 = \underline{\quad ? \quad}$
59. $2 \times 10_6^2 + 3 \times 10_6^1 + 2 = \underline{\quad ? \quad}$
60. The symbol m'_k denotes the k th moment about the origin.
61. a'_1, a'_2, \dots, a'_n are inverses of $\underline{\quad ? \quad}$.
62. From the given information we can deduce that $y^{4'}(0) = 3^2 \cdot 4$.
63. In x^3 , the 3 is the superscript.
64. Prove that $(ab)^x = a^x b^x$ if k_1 and k_2 denote any real numbers such that $a = 2^{k_1}$ and $b = 2^{k_2}$. (Use the laws of exponents for 2^x where $(ab)^x = 2^{k_1 x} \cdot 2^{k_2 x}$.)
65. From the given information prove that the product of $e^{i\theta_1}$ and $e^{i\theta_2}$ equals $e^{i(\theta_1 + \theta_2)}$.
66. Take the natural logarithm of $e^{x_1 + x_2 + x_3 + \dots + x_n}$.
67. In what situations is the factorization $a = 2^{a_0} 3^{a_1} 5^{a_2} \dots$ regarded as extending indefinitely?
68. Name the prime factors of n^2 if $n = p_1^{x_1} p_2^{x_2} \dots p_k^{x_k}$.
69. Practice writing the following symbols: $x^{p_{qr}}, x^{p_{qr}}, x^{p_{qr}}, x_{2p}, x_{p_{qr}}, x_{p_{qr}}, x_{p_{qr}}$.
70. We have learned patterns that are often called laws. For example
 (1) $a + b = b + a$ is the commutative law for addition.
 (2) $a \times b = b \times a$ is the commutative law for multiplication.
 a. Use symbols to express the associative laws for addition and multiplication.
 b. Use symbols to express the distributive law.
71. a. In the sequence 10, 5, 0, —5, \dots , which term is —75?
 b. Find the 9th term of $(x - y)^9$.
- Factor.
- | a | b |
|-----------------------|-----------------------|
| 72. $x^2 - x - 12$ | $x^2 + 12x + 35$ |
| 73. $6x^2 + 11x - 35$ | $6x^{2a} + 11x^a + 4$ |
| 74. $9x^2 - 16y^2$ | $ax^3x^2 + a^2$ |

LESSON 7

FRACTIONS AND FRACTION INDICATORS

§70. General Principles: Although the numerator and denominator of a fraction may be separated by a horizontal or a diagonal fraction line, unless otherwise stated the fraction should be transcribed linearly so that the numerator, the fraction line, and the denominator are written horizontally across one braille line.

The spacing before and after a fraction is subject to the spacing rules for the signs preceding or following the fraction. In addition, no space should be left between a fraction and a letter, a numeral, a sign of grouping, a braille indicator, or another fraction when these items are unspaced in print and are part of the same expression.

§71. Simple Fractions: A simple fraction is one in which neither the numerator nor the denominator is a fraction. A fraction is also considered a simple fraction when its numerator or denominator contains fractions at the superscript or subscript level only.

$$\frac{3}{4} \quad \frac{a+b}{c+d} \quad \frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}} \quad x/y \quad \frac{a+b}{c+d}$$

Simple Fraction Indicators

Opening

• •

• •

• •

Closing

Fraction Lines Used With Simple Fractions

Horizontal Simple Fraction Line

• •

Diagonal Simple Fraction Line

§72. Use of Simple Fraction Indicators:

a. The opening and closing simple fraction indicators must be used to enclose a simple fraction whose numerator and denominator are separated by a horizontal fraction line.

The horizontal fraction line must be shown by $\frac{\cdot}{\cdot}$.

$$(1) \quad \frac{3}{4}$$

$$(2) \quad \frac{a + b}{c + d}$$

$$(3) \quad \frac{\Delta y}{\Delta x}$$

(4) $x \frac{1}{8}$



(5) $V = \frac{1}{3} \pi r^2 h$



(6) $\frac{1}{4} + \frac{3}{4} - \frac{1}{2} = \frac{1}{2}$



(7) $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$



(8) $\frac{-1}{2} + \frac{-3}{4}$



(9) $x^{-\frac{1}{2}}$



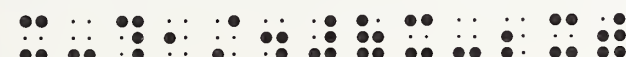
(10) $x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}}$



(11) $\frac{x^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1}$



(12) $x - \frac{1}{4}(x - 2x)$



(13) $\left(\frac{3}{2}a + \frac{1}{2}b \right)$



(14) $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$



$$(15) \quad \frac{1}{2} \frac{n-1}{n} = \frac{n-1}{2n}$$



$$(16) \quad \begin{array}{r} \$15.25 \\ \$25.50 \end{array}$$



$$(17) \quad \$\frac{3}{5} \div \$\frac{1}{5} = \$3$$



$$(18) \quad \frac{3}{4}\phi - \frac{1}{4}\phi = \frac{1}{2}\phi$$



$$(19) \quad \frac{1}{4}\% + \frac{1}{2}\% = \frac{3}{4}\%$$



$$(20) \quad \frac{5''}{8} + \frac{2''}{8} = \frac{7''}{8}$$



$$(21) \quad \frac{6}{8} = \frac{\quad}{4}$$



$$(22) \quad \frac{1??}{1000} = \frac{1}{10}$$



b. Simple fraction indicators must be used to enclose a simple fraction whose numerator and denominator are separated by a diagonal line when the numerator and denominator are printed at different levels of writing on either side of the diagonal line or in a different type size from that normally used for such fractions throughout the text. The diagonal fraction line must be shown by



$$(1) \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$



$$(2) \quad \text{Reduce } \frac{18}{24} \text{ to lowest terms.}$$



$$(3) \quad 3 \times/y$$



(4) $2 \frac{1}{y}$



(5) $y \frac{1}{5}$



§73. Nonuse of Simple Fraction Indicators: Simple fraction indicators must not be used to enclose a simple fraction whose numerator and denominator are separated by a diagonal line when the numerator and denominator are printed at the same level of writing on either side of the diagonal line and in the same type size as the rest of the mathematical expression.

(1) $\frac{1}{2}$



(2) $\frac{2a}{3b}$



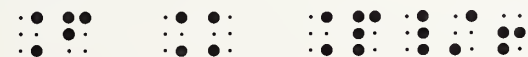
(3) $a + \frac{b}{c} + d$



(4) $\frac{5}{8} + \frac{3}{8} = \frac{8}{8} = 1$



(5) $\phi = \frac{\pi}{6}$



(6) $\frac{(x + y)}{(x - y)}$



(7) $x^{\frac{1}{2}} + \frac{1}{y^{\frac{1}{2}}} + 1$



(8) $x^{1/2} - y^{1/2}$



(9) $\frac{1}{4}\phi + \frac{1}{2}\phi = \frac{3}{4}\phi$



(10) $\frac{3}{8''} - \frac{2}{8''} = \frac{1}{8''}$





§74. **Mixed Numbers:** A mixed number is an expression composed of a whole number followed by a simple fraction whose numerator and denominator are both numerals. Numerals in a mixed number may be represented by omission signs. An expression is not a mixed number if it contains any letters, even though the expression appears to be in the form of a mixed number.

$1\frac{1}{2}$
 $1\frac{1}{2}$

Mixed Number Fraction Indicators

Opening	
Closing	

Fraction Lines Used With the Fractional Part of a Mixed Number

Horizontal Simple Fraction Line		
Diagonal Simple Fraction Line		

§75. **Use of Mixed Number Fraction Indicators:** The opening and closing mixed number fraction indicators must always be used to enclose the fractional part of a mixed number. The appropriate braille symbol must be used to represent the horizontal or diagonal fraction line shown in print.

(1)

$1\frac{1}{2}$



(2)

$1\frac{1}{2}$



(3)

$64\frac{325}{1000}$


(4)

$1\frac{1}{2} + 2\frac{2}{3} = 4\frac{1}{6}$


(5)

$1\frac{1}{2} + 2\frac{2}{3} = 4\frac{1}{6}$


(6)

$(3\frac{1}{2} \times 2) + (\frac{1}{2} \times 3\frac{1}{2})$


(7)

$2(\frac{1}{4}) + 2\frac{3}{4} = 3\frac{1}{4}$


(8) $2\frac{1}{2}x + 4 = 14 + \frac{1}{2}x$



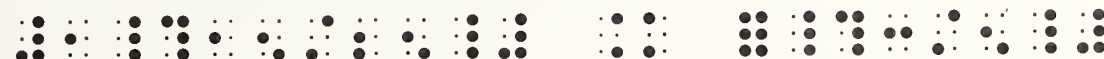
(9) $1\frac{3}{4}\% + 4\% = 5\frac{3}{4}\%$



(10) $8\frac{1''}{2} + 7\frac{5''}{6}$



(11) $1\frac{15}{25} = ? \frac{3}{5}$



(12) $7/4 = 1 \frac{3}{4}$



§76. Complex Fractions: A complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers. A fraction is not a complex fraction if the only fractions it contains are at the superscript or subscript level; such a fraction is a simple fraction.

$$\frac{\frac{1}{4}}{\frac{3}{4}} \quad \frac{\frac{a}{b} - \frac{c}{d}}{\frac{a}{b} + \frac{c}{d}} \quad \frac{1/4}{3/4} \quad \frac{4\frac{3}{4}}{5} \quad \frac{1}{2} \bigg/ \frac{3}{4}$$

Complex Fraction Indicators

Opening



Closing



Fraction Lines Used With Complex Fractions

Horizontal Complex Fraction Line



Diagonal Complex Fraction Line



§77. Use of Complex Fraction Indicators:

a. The opening and closing complex fraction indicators must be used to enclose a complex fraction, and the main complex fraction line must be represented by its appropriate braille symbol.

(1)
$$\frac{\frac{1}{4}}{\frac{3}{4}}$$

(2) $\frac{\frac{a}{b}}{c}$

(3) $\frac{\frac{\pi}{8}}{2\pi}$

(4)
$$\frac{\frac{3-1}{4+3}}{\frac{2-1}{3+5}}$$

$$(5) \quad \frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} - \frac{7}{9}}$$

$$(6) \quad \frac{\begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}{1 \cdot 2 \cdot 3}$$



(7) $\frac{\frac{1}{6}}{5} + \frac{5}{\frac{1}{6}}$

(8) $\frac{1/3 + 1/4}{4/5 - 1/2}$

$$(4) \frac{\frac{x}{x-1} - 1^2}{\frac{x}{x+1} + 1^2}$$



$$(5) r^2/4 - s^2/9 + t^2$$


$$(6) x^{1/2}/2$$


$$(7) \frac{y_2 - y_1}{x_2 - x_1}$$


$$(8) a_1/b + a_2/b^2 + a_3/b^3 + \dots$$


§79. Fractions and the Ellipsis and Long Dash: No space should be left between an opening or closing fraction indicator and an ellipsis or long dash in the numerator or denominator of a fraction. However, a space must be left between a fraction line and an ellipsis or long dash. A space must also be left between a fraction and an ellipsis or long dash preceding or following the fraction.

$$(1) \frac{\dots \times 5}{2 \times 10} = \frac{15}{20}$$


$$(2) \frac{2 + 4 + 6 + \dots}{1 + 3 + 5 + \dots}$$


$$(3) \frac{\text{---} \times 3}{3 \times 4} = \frac{12}{12}$$


$$(4) \frac{1}{10} \dots \frac{10}{10}$$

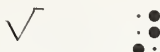

$$(5) \frac{1}{10} \text{---} \frac{10}{10}$$


RADICALS AND RADICAL INDICATORS

§82. **Radicals:** The symbol $\sqrt{\quad}$ is called a *radical sign*. A *vinculum*, or horizontal bar, is generally used with the radical sign $\sqrt{\quad}$ to show the extent to which the radical sign applies. The expression under the radical sign is called the *radicand*. A figure placed to the left and slightly above the radical sign is called the *index* of the radical.

$$\sqrt{4} \qquad \sqrt[3]{x+y}$$

Radical Sign



Radical Indicators

Index-of-Radical Indicator



Order-of-Radical Indicators

First Inner Radical



Second Inner Radical





Third Inner Radical



Termination Indicator



§83. Simple Radicals:

a. When a radical has a *vinculum*, the radical sign  must be placed before the radicand, and the termination indicator  must be placed after the radicand. However, the termination indicator must not be used when the radical sign occurs without a radicand, or when the *vinculum* is not shown in print.

(1) $\sqrt{64}$



(2) \sqrt{a}



(3) $\sqrt{s(s-a)(s-b)(s-c)}$



(4) $\sqrt{\frac{a}{b} + \frac{b}{a}}$



(5) $\sqrt{a/b}$



$$(11) \quad \frac{\sqrt{2} - \sqrt{\frac{1}{3}}}{\sqrt{3} - \sqrt{\frac{1}{2}}}$$

$$(12) \quad p^{\sqrt{q}} + r$$

$$(13) \quad \sqrt{x-1} + \sqrt{2x} = 1$$

c. When a radical has an index, the index-of-radical indicator $\sqrt[n]{}$ and the index of the radical must precede the radical sign.

(1) $\sqrt[3]{27}$

$$(2) \sqrt[n]{a}$$

(3) $\sqrt{a+b} \sqrt{x-y}$

(4) $\sqrt[4]{729} + \sqrt[6]{27}$

(5) $3\sqrt[3]{125} \cdot 3\sqrt[5]{2}$

$$(6) \quad \sqrt[5]{c} \sqrt[5]{d} = \sqrt[5]{cd}$$

$$(7) \quad \sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$$

§84. **Nested Radicals:** When radicals are nested one within the other, the appropriate number of order-of-radical indicators must be used to show the depth of each *inner* radical.

If an inner radical has no index, the appropriate order-of-radical indicator must be placed before its radical sign. If an inner radical has an index, the appropriate order-of-radical indicator must be placed before the index-of-radical indicator.

The order-of-radical indicator used before an inner radical must also be used before its corresponding termination indicator. When more than one radical is completed at the same point, the radicals are terminated, beginning with the innermost radical.

(1) $\sqrt{x + \sqrt{x + y} + z}$

(2) $\sqrt{-\frac{1}{2} - i\frac{\sqrt{3}}{2}}$

(3) $\sqrt{\sqrt{13} + \sqrt{15} + \sqrt{117}}$

(4) $\sqrt{1 - \sqrt{a - b}} \times \sqrt{1 + \sqrt{a - b}}$

(5) $\sqrt{x + \sqrt{y + \sqrt{z}}}$

(6) $\sqrt{\sqrt[3]{16}} = \sqrt[3]{\sqrt{16}}$

(7) $\sqrt{b \sqrt[3]{b} \sqrt{b}}$

(8) $\sqrt{\sqrt[3]{\sqrt[4]{10}}}$

(9) $\sqrt[4]{\sqrt[3]{\sqrt[5]{abc}}}$

§85. **Radicals and the Baseline Indicator:** When a level indicator is in effect, the baseline indicator $\cdot\cdot$ must be used before a radical sign or radical indicator belonging to a radical on the baseline of writing. However, the baseline indicator must not be used after a numeric subscript not requiring a subscript indicator.

(1) $(r^2\sqrt{r})^2$

$$(2) \sqrt{x^2 + y^2}$$



$$(3) \sqrt{a^2} \sqrt{b^4} \sqrt{c} = ab^2\sqrt{c}$$



$$(4) (s^2 \sqrt[3]{s^4})^2$$



$$(5) \sqrt[3]{x^2} \sqrt{64x^6}$$



$$(6) \sqrt{\sqrt[3]{\sqrt[4]{\sqrt[5]{b^{48}}}}}$$



$$(7) \sqrt{x_1 + y_2}$$



$$(8) \sqrt{x_1 + \sqrt{x_2}}$$



§86. Radicals and the Ellipsis and Long Dash: When an ellipsis or a long dash occurs within a radical, no space should be left between the ellipsis or long dash and the termination indicator or order-of-radical indicator. However, a space must be left between the radical sign and an ellipsis or long dash. A space must also be left between a radical and an ellipsis or long dash preceding or following the radical.

$$(1) \sqrt{a + b + c + \dots}$$



$$(2) \sqrt{x + \sqrt{x + \sqrt{\dots}}}$$



$$(3) \sqrt{4} \dots \sqrt{64}$$



§87. Radicals and Abbreviations: When an abbreviation occurs within a radical, no space should be left between the abbreviation and the termination or order-of-radical indicator following it. However, a space must be left between a radical sign and an abbrevia-

tion. A space must also be left between a radical and an abbreviation preceding or following the radical.

(1) $\sqrt{9 \text{ ft}}$



(2) $\sqrt{\text{ft.}}$



(3) $2\sqrt{12 \text{ sq. in.}}$



§88. **Enclosed Lists and Punctuation With Radicals:** Radicals may be part of an enclosed list. The radical sign and radical indicators must be punctuated mathematically.

(1) $(\sqrt{9}, 3, \sqrt{4}, 2\sqrt{6})$



(2) Simplify: $\sqrt{2}, \sqrt{18}, \sqrt{24}$.



(3) " $\sqrt{\quad}$ " means "square root."



FORMAT (CONTINUED)

§89. **Displayed Expressions:** When a mathematical expression or other material is set apart from the body of the text by skipped lines, centering, special indentation, etc., such displayed expressions must be transcribed in the following format:

a. A line must not be skipped above or below a displayed expression unless the expression precedes or follows a heading, or unless a blank line is required under other provisions of the code.

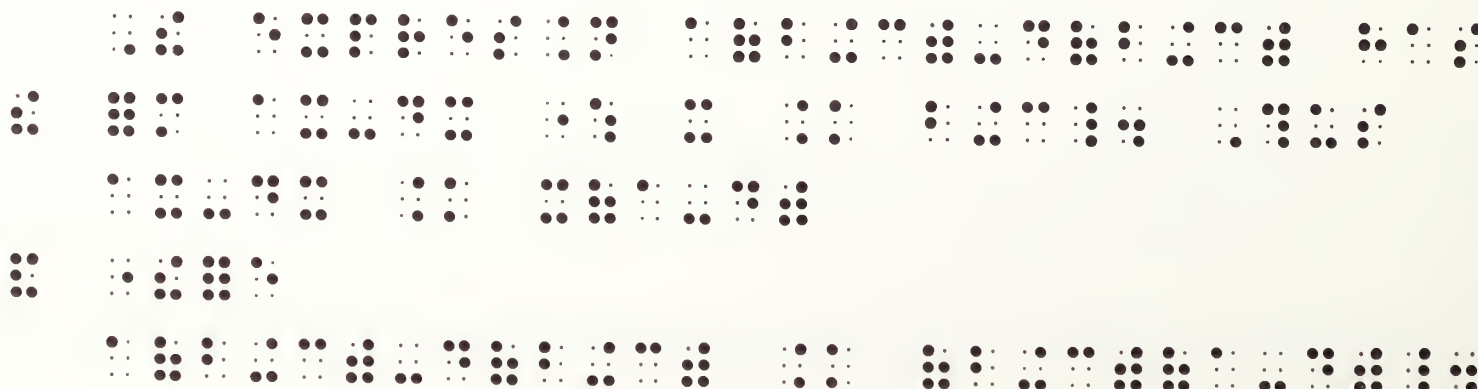
b. When a displayed expression occurs in unitemized explanatory portions of the text, it must begin in cell 3, and its runovers must begin in cell 5.

(1) The expression $a(b + c) - d(b + c)$ has the form $ax - dx$ where $x = b + c$. Thus

$$ax - dx = x(a - d)$$

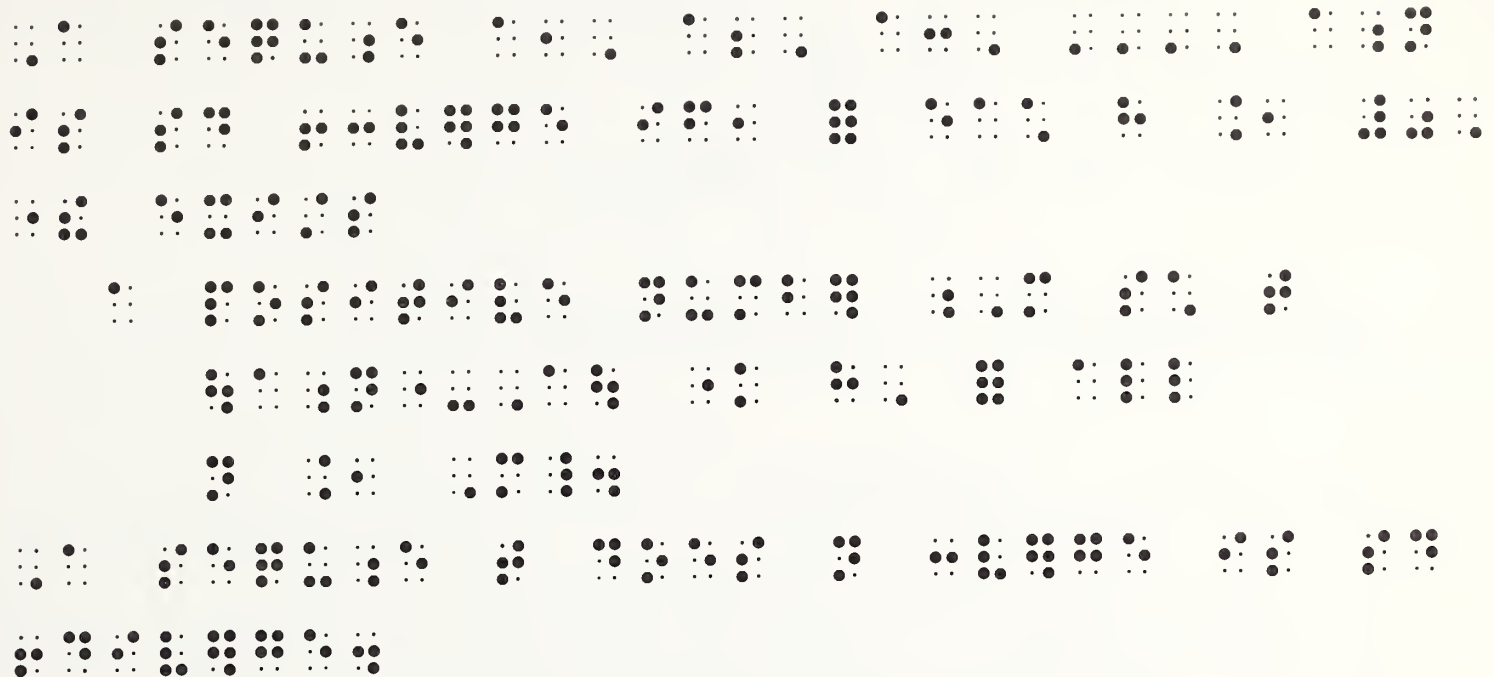
and therefore

$$a(b + c) - d(b + c) = (b + c)(a - d).$$



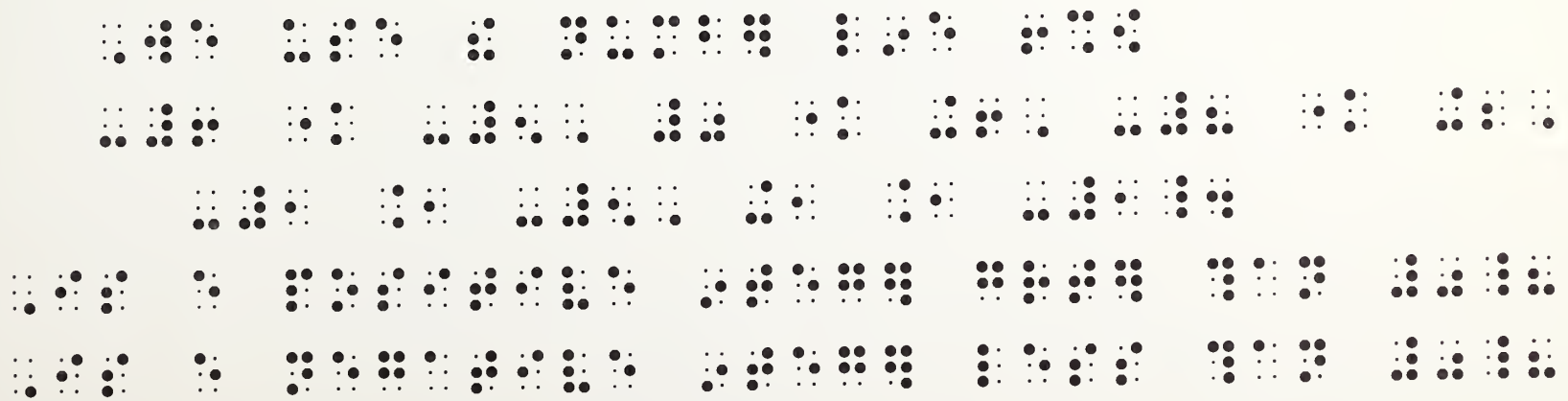
- (2) A sequence $a_1, a_2, a_3, \dots, a_n$ is said to converge if, for each $h > 0$, there exists a positive number M such that $|a_n - A| < h$, for all $n > M$.

A sequence that does not converge is said to diverge.



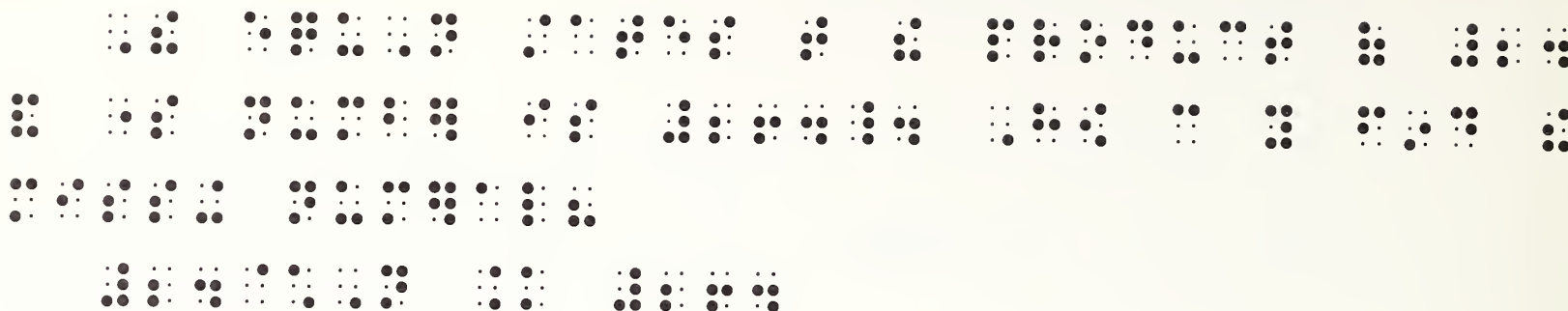
- (3) We use the number line to show
 $-6 < -5, 0 < +6, -8 < +2, -1 > -5, +1 > -1.$

Is every positive integer greater than 0? Is every negative integer less than 0?



- (4) The equation states that the product of 24 and some number is 264. How can you find the missing numeral?

$$24 \times N = 264$$



c. When a displayed expression occurs with an itemized text containing no subdivisions, it must begin in cell 5, and its run-overs must begin in cell 7.

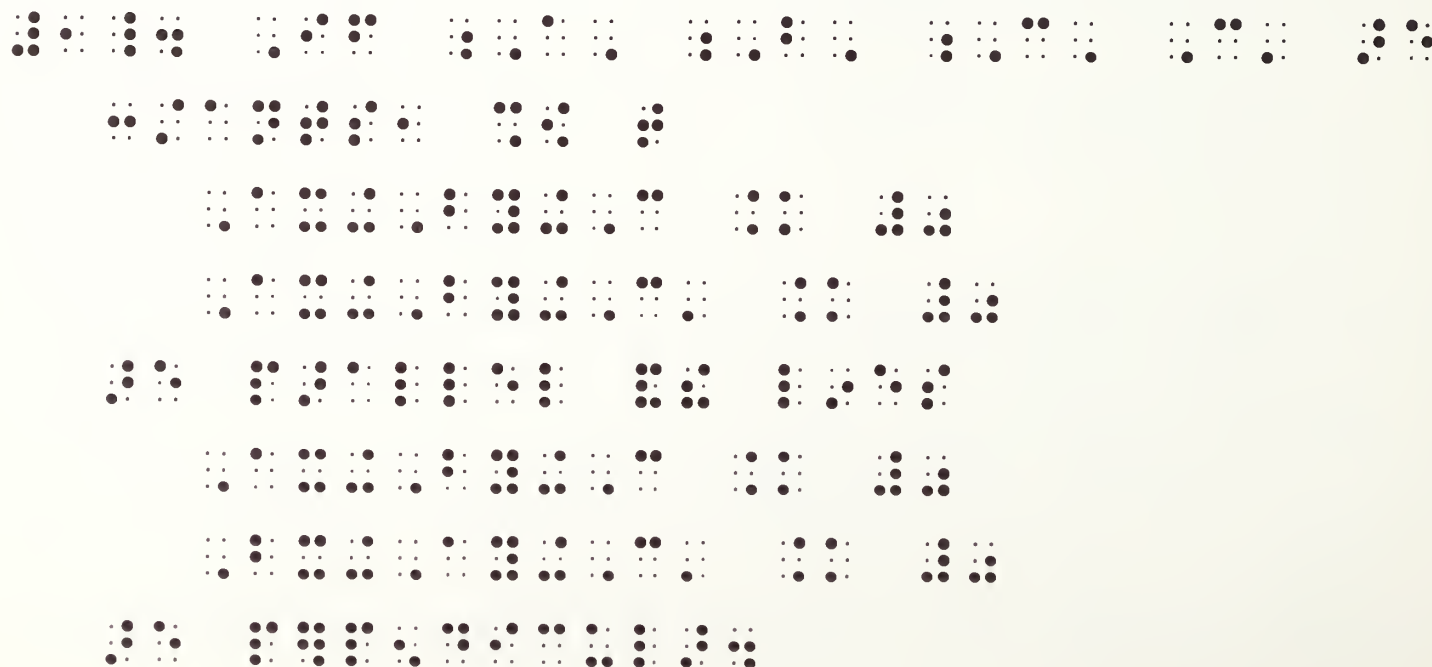
- (1) 1. If A, B, C, C' are constants, show that

$$\begin{aligned} Ax + By + C &= 0 \\ Ax + By + C' &= 0 \end{aligned}$$

are parallel and the lines

$$\begin{aligned} Ax + By + C &= 0 \\ Bx + Ay + C' &= 0 \end{aligned}$$

are perpendicular.

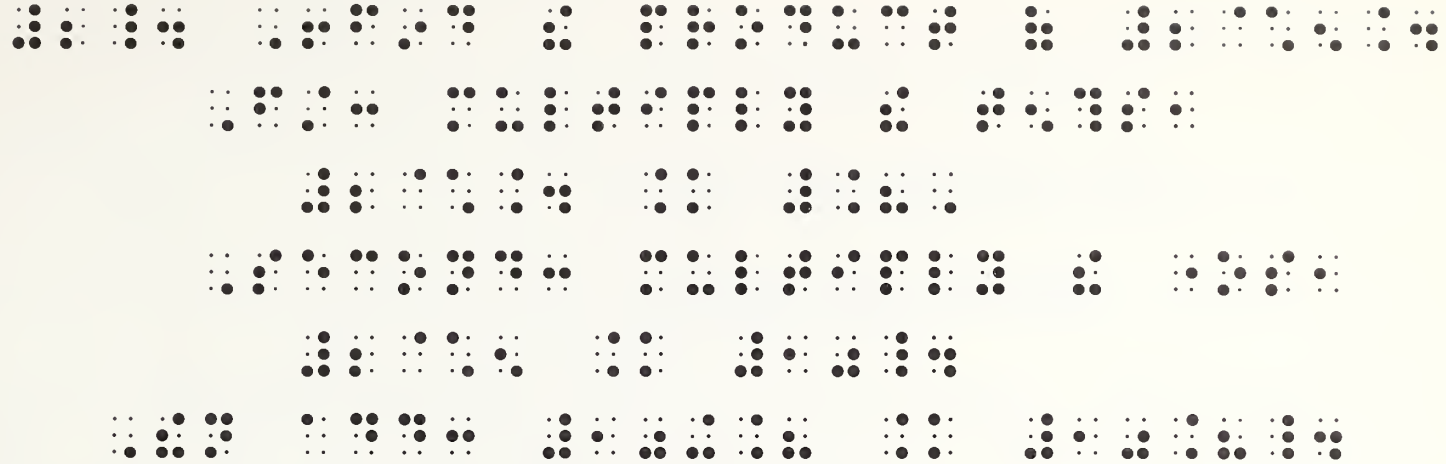


(2) 2. To find the product of 2×5.4

First: multiply the tenths, $2 \times .4 = .8$,

Second: multiply the ones, $2 \times 5 = 10$.

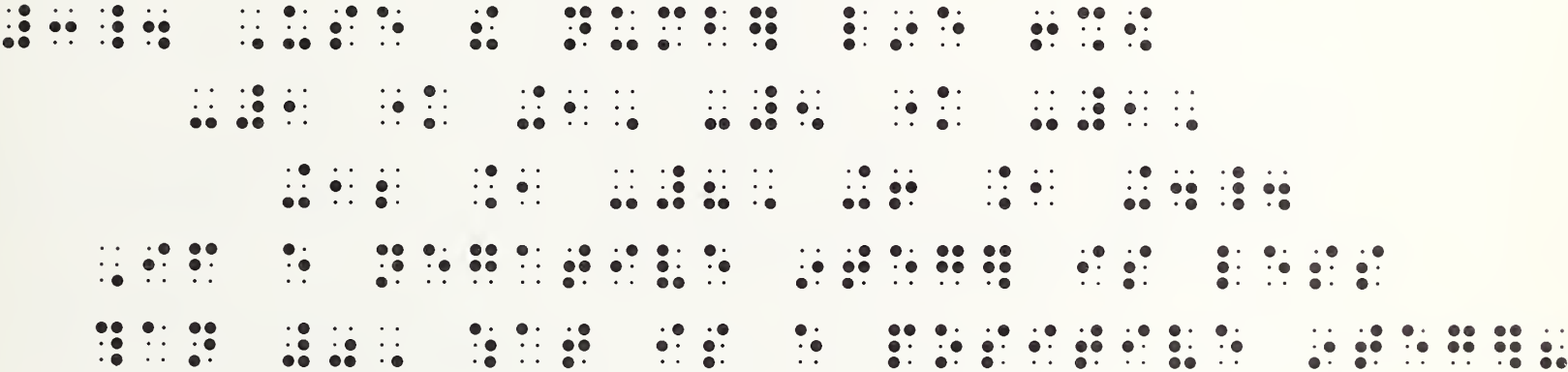
Then add: $10 + .8 = 10.8$.



(3) 3. Use the number line to show

$$-1 < +1, -5 < -1, +12 > -8, +6 > +4.$$

If every negative integer is less than 0,
what is every positive integer?



d. When a displayed expression occurs with an itemized text containing subdivisions, it must begin in cell 7, and its runovers must begin in cell 9.

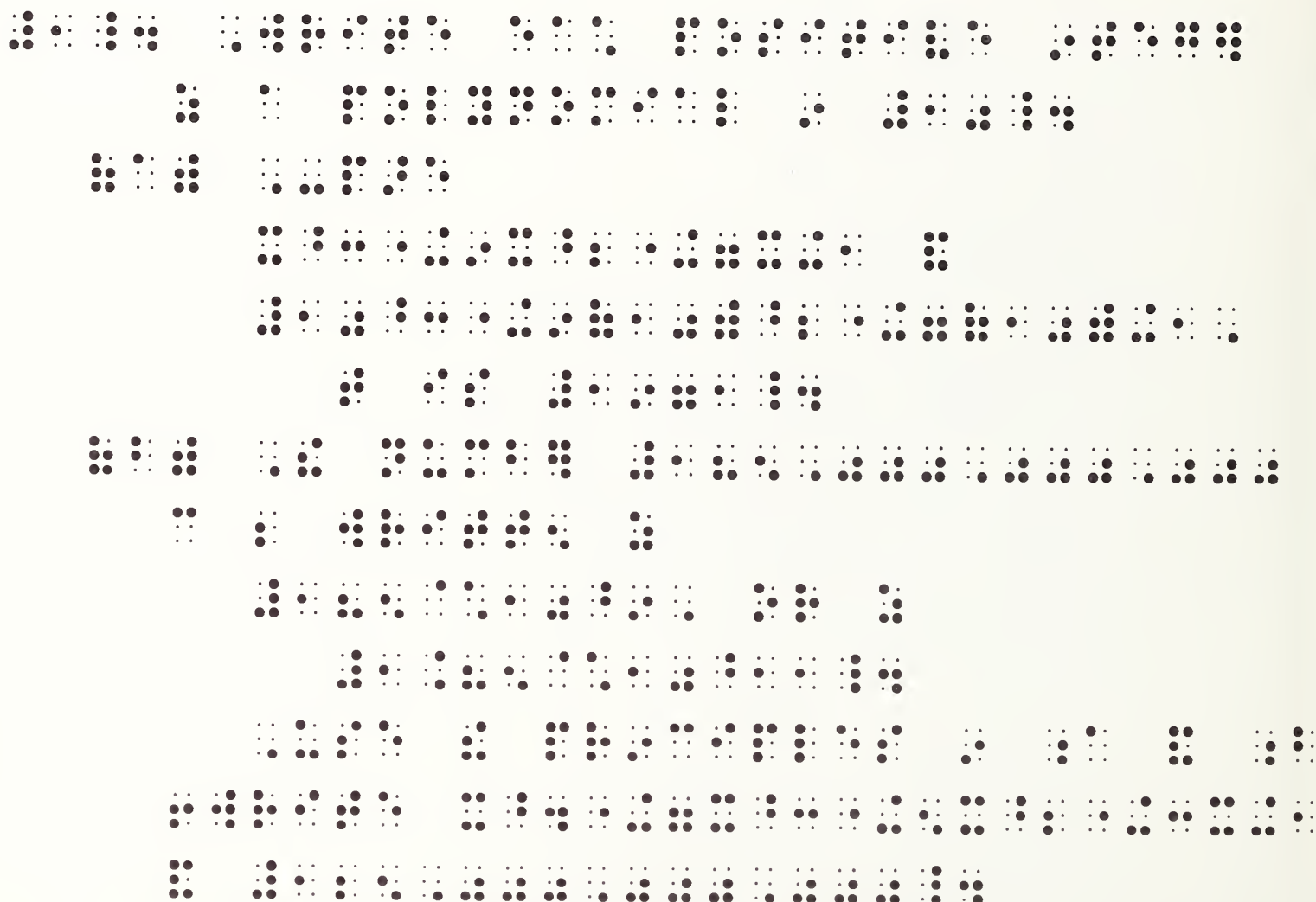
- (1) 1. Write each positive integer as a polynomial in 10.

(a) Compare

$$x^3 + 9x^2 + 7x + 1 \text{ and } 10^3 + 9(10)^2 + 7(10) + 1, \text{ that is } 1971.$$

- (b) The number 185,000,000,000 can be written as 185×10^9 , or as 1.85×10^{11} .

Use the principles in a and b to write $x^4 + 7x^3 + 5x^2 + 3x + 1$ and 125,000,000,000.



e. When a number or letter is used to identify a displayed expression, it must begin in the appropriate cell for displayed material in accordance with the rules of a—d above. If such numbers or letters occur to the right of the expression in print, they must be placed at the left of the expression in braille, and a transcriber's note concerning the change in position must be incorporated at the beginning of the first volume.

A page reference to a displayed expression must immediately follow that expression.

- (1) Two basic laws of arithmetic are the

commutative law for addition

$$a + b = b + a, \quad (1)$$

and the *commutative law for multiplication*

$$\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}. \quad (2)$$

- (2) 2. Use the formula

$$a_n = a_1 + (n - 1)d \quad (12.3)$$

to find a_n and n when $a_1 = 1$, $d = 7$, $s_n = 204$.

- (3) 3. Give two examples illustrating

- (a) The associative law for addition

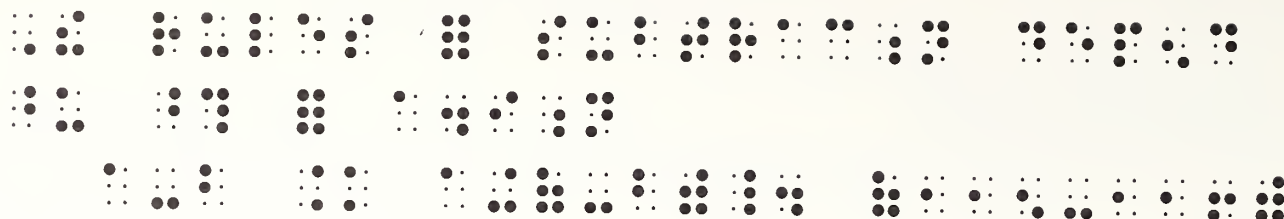
$$(a + b) + c = a + (b + c). \quad (4)$$

- (b) The associative law for multiplication

$$(a \times b) \times c = a \times (b \times c). \quad (5)$$

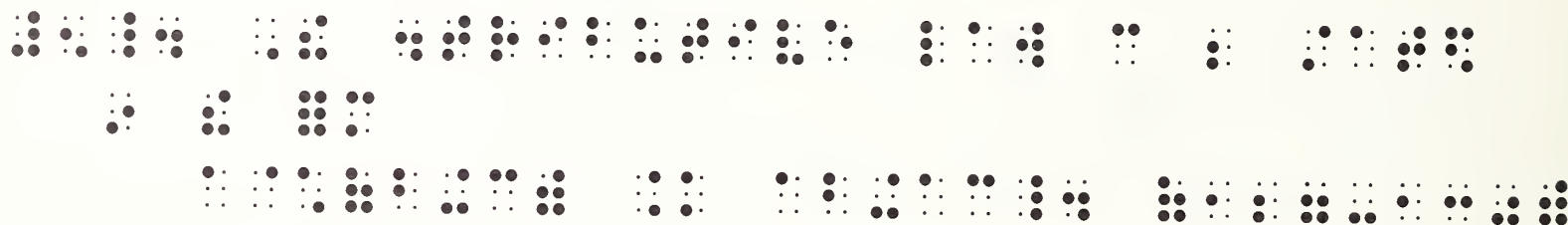
- (4) The rules for subtraction depend upon those for addition

$$a - b = a + (-b). \quad (115-116)$$



- (5) 5. The distributive law can be stated in the form

$$a \times (b + c) = ab + ac. \quad (127-130)$$



HOMework

Prepare the following homework for submission to your teacher. Note: Page 130 contains material requiring a transcriber's note at the beginning of a braille volume. This note should be placed on a separate page at the beginning of the homework. Proofread carefully.

EXERCISE 7

- Which fractions are unlike any other fraction: $\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \frac{4}{8}, \frac{7}{8}, \frac{15}{16}$.
- Ted subtracted $\frac{1}{2}$ from $\frac{17}{18}$ and got $\frac{16}{16}$. Why isn't this a reasonable answer?
- How many $\frac{2}{3}$'s are there in $\frac{5}{6}$?
- Find the number of $\frac{1}{16}$'s there are in a $\frac{1}{2}$ -foot line.
- Find the value of $\frac{E}{r + R}$ when $E = \frac{5}{2}$, $r = \frac{3}{4}$, and $R = \frac{7}{8}$.
- If $\frac{p}{q}$ and 0 are not equal, show that $a^{\frac{p}{q}} = b^{\frac{p}{q}}$ if and only if $a = b$.
- What rule is illustrated by: $\left| \frac{c}{d} \right| = \frac{|c|}{|d|}$?
- Estimate: $\frac{7987 \times 803 \times 0.061}{\pi}$.
- Explain why the formula $A = \frac{1}{2}bh$ can be written as $A = \frac{bh}{2}$.
- If $\frac{n}{\$90} = \frac{120}{100}$, what does n equal?

11. Does $\frac{1}{2}$ of \$ $\frac{4}{5} = 40¢$?

12. How many $\frac{1}{2}''$ are there in $1''$?

13. If $\frac{a}{b}$ and $\frac{c}{d}$ represent any two fractional numbers, then does

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}?$$

14. Solve and check:

(a) $\frac{5}{8} + \frac{1}{2} + \frac{3}{4} = ?$

(e) $\frac{+1}{8} + \frac{+3}{4} = ?$

(b) $\left(\frac{8}{7} \times \frac{6}{11}\right) \times \frac{7}{8} = ?$

(f) $\frac{3}{4}\% + \frac{1}{4}\% + \frac{1}{2}\% = ?\%$

(c) $\frac{4}{8} = \frac{?}{2}$

(g) $\frac{\dots \times 20}{\dots \times 30} = \frac{60}{90}$

(d) $\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \times \frac{6}{5} = ?$

15. Simplify:

(a) $5\frac{x}{y} + 3\frac{x}{y}$

(c) $\frac{1}{2} \left[\left(\frac{1}{2} - \frac{1}{3} \right) \div \frac{1}{6} \right]$

(b) $\frac{x+a}{ax} - \frac{1}{a} = \frac{a}{x+a}$

(d) $(a^{\frac{1}{2}} - b^{\frac{1}{2}})^2$

16. Sketch the graph of $\frac{x^2}{9} + \frac{y^2}{36} = 1$.

17. What number does $5 \times \frac{1}{10^3}$ name?

18. Express $\frac{5a^{\frac{3}{4}}}{b^{\frac{5}{4}}}$ in radical form.

19. Use the formula $\frac{T_1}{T_2} = \frac{T_3}{T_4}$ to find the value of T_3 if $T_1 = 150$, $T_2 = 96$, and $T_4 = 144$.

20. Show $p\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ equidistant from $m(x_1, y_1)$ and $n(x_2, y_2)$.

21. Use $\frac{1 \text{ mi}}{1400 \text{ mi}}$ to find the relative error of the measurement 1 mi. in 1400 mi.

22. $\frac{5}{6} \text{ lb} + \frac{7}{8} \text{ lb} + \frac{1}{4} \text{ lb} = \underline{\hspace{1cm}}$

23. $\frac{4}{8} \text{ yd.} \cdot \frac{3}{4} \text{ ft.} = ? \text{ in.}$

24. Reduce $\frac{16}{32}$ and $\frac{14}{18}$ to lowest terms.

25. If $\frac{a}{b}$ and $\frac{c}{d}$ represent any two fractional numbers, then does $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$?

26. In $3a/b$ and $r\frac{1}{4}$, the 3 and r are coefficients of the fraction. Explain.
27. Find the 7th term of $1/2, 1/4, 1/8, \dots$.
28. Fractions may be expressed in many ways. For example, $2/3 = 8/12; 1/1 = 2/2 = 17/17 = x/x$.
29. Find the correct answer:
- (a) $7/15 \times 5/14 \times 1/8$ (d) $7/8' - 4/8'$
 (b) $5/6 \div 1/3$ (e) $(25)^{3/2} - (-343)^{2/3}$
 (c) $3/4\phi + 1/2\phi$
30. Express $\frac{6x^{5/6}}{x^{7/6}}$ in radical form.
31. Use Table 5.5 to compute $F_{1/4}$, $F_{2/4}$, and $F_{3/4}$.
32. Find $D_t\theta]_{\theta=\pi/2}$ in the given problem.
33. An object travels 88 ft./sec. at 60 mi./hr. How far will it travel in 1 sec. when the speed is 750 mi./hr.?
34. What is the sum of $4\frac{1}{4}$, $3\frac{3}{4}$, and $10\frac{1}{2}$?
35. Insert 6 arithmetic means between $7\frac{1}{2}a$ and $\frac{1}{2}a$.
36. Find the correct answer:
- (a) $2\frac{1}{3} \times 3\frac{1}{2} \times 1\frac{1}{6}$ (b) $3\frac{2}{3} + 1\frac{1}{2} + \frac{4}{5}$
 (c) $-1\frac{1}{7} \times (-2\frac{1}{3})$ (d) $(3\frac{1}{3})(43\frac{1}{6})$
 (e) $5\frac{4}{3} = 6\frac{?}{3}$ (f) $8\frac{4}{3} = ?\frac{1}{3}$
 (g) $3\frac{1}{3}y = 13\frac{1}{3}$ (h) $1\frac{7''}{8} + 2\frac{1''}{4}$
37. Find: $5\frac{3}{4} \div 2\frac{2}{3}; 14\frac{7}{8} \div 3\frac{1}{16}$.
38. 1 cu. ft. holds about $7\frac{1}{2}$ gal. How many gal. are in a tank $1\frac{2}{3}$ ft \times $1\frac{1}{2}$ ft \times 1 ft?
39. Find the premium for a $1\frac{1}{2}$ -yr. policy at the yearly rate of 24¢ per \$100.
40. Is there a 1-to-1 correspondence between the members of
 $\{0, \frac{1}{2}, 1, 1\frac{1}{2}\}$ and
 $\{2 - 2, 1 \div \frac{1}{2}, 4 - 3, 1 + \frac{1}{2}\}$?

Simplify:

41. $\frac{\frac{1}{2}}{\frac{3}{4}}$

42. $\frac{\frac{15}{22}}{\frac{47}{88}}$

43. $\frac{\frac{4}{7}}{2}$

$$44. \frac{\frac{55 + 42}{15}}{\frac{34 + 45}{18}}$$

$$45. \frac{\frac{x + y}{x - y}}{\frac{a}{b}}$$

$$46. \frac{19}{32 \times \frac{1}{2}}$$

$$47. \frac{\frac{1}{8} + \frac{3}{4}}{7}$$

$$48. \frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} - \frac{7}{6}}$$

$$49. \frac{1 + \frac{a}{b}}{1 - \frac{a^2}{b^2}}$$

$$50. \frac{a + \frac{a}{b + c}}{c + \frac{c}{b + c}}$$

$$51. \frac{1}{\frac{a}{b}} \times \frac{b}{b}$$

$$52. \frac{3/5}{6}$$

$$53. \frac{11/3 + 14/5}{17/9 + 5/2}$$

$$54. \frac{33\frac{1}{3}}{100}$$

$$55. \frac{2\frac{1}{2}}{5\frac{1}{3}}$$

$$56. \frac{2 \times 12\frac{1}{2}}{2 \times 100}$$

$$57. \frac{x}{y^{\frac{1}{4}}}$$

$$58. \frac{3}{5} \div \frac{7}{9}$$

$$59. \text{ Find the square units of the region in Fig. 5 whose area is } \frac{25^{\frac{3}{4}} - 6^{\frac{3}{4}}}{\frac{3}{4}}.$$

60. Simplify these radicals:

$$(a) \sqrt{63}$$

$$(b) \sqrt{27}$$

$$(c) \sqrt{\frac{a + b}{a - b}}$$

$$(d) (r^2\sqrt{r^3})^3$$

$$(e) \sqrt{80a^6b^2}$$

$$(f) \sqrt{48x^3y^3}$$

61. Find:

$$(a) \sqrt{75} - \sqrt{48}$$

$$(b) 2\sqrt{108} - 3\sqrt{27}$$

$$(c) (7 - 2\sqrt{5})(7 + 2\sqrt{5})$$

$$(d) \sqrt{2}\sqrt{10}$$

62. Simplify:

$$(a) \frac{2 + \sqrt{3}}{\sqrt{3}}$$

$$(b) \frac{4\sqrt{6} - 3\sqrt{2}}{7\sqrt{3} + 2\sqrt{5}}$$

$$(c) \frac{2 - \sqrt{\frac{1}{4}}}{3 - \sqrt{\frac{1}{2}}}$$

63. Find the sum of 7 terms of $3\sqrt{2}$, 6, $6\sqrt{2}$, ...

64. Find equivalent numbers for the following:

$$(\sqrt{8}, 4, \sqrt{16}, 2\sqrt{2}).$$

65. The symbol $\sqrt{\quad}$ means "square root."

66. Find: $\sqrt{(30)} \times \sqrt{(10)} \times \sqrt{(10)}$.

67. Simplify:

(a) $\sqrt[3]{72}$

(b) $\sqrt[3]{54}$

(c) $\sqrt[6]{(a + b)^3}$

(d) $\frac{\sqrt[3]{9}}{\sqrt[3]{18}}$

68. Find:

(a) $\sqrt[3]{16} + \sqrt[4]{162}$

(b) $\sqrt[5]{8} \cdot \sqrt[5]{2} \cdot \sqrt[5]{2}$

(c) $(3\sqrt[3]{a + 2})^3$

(d) $\sqrt[3]{7} \sqrt[3]{7}$

(e) $\sqrt[4]{8r^2s^3t^5} \sqrt[4]{4r^6st^2}$

69. Express each of the following as an integer or as a single radical.

(a) $\sqrt{\sqrt{a}}$

(b) $\sqrt[3]{\sqrt{729}}$

(c) $\sqrt[3]{r\sqrt{r}}$

(d) $\sqrt{\sqrt{15} + \sqrt{17} + \sqrt{117}}$

(e) $\sqrt{\sqrt[3]{64}}$

(f) $\sqrt{\sqrt[3]{\sqrt[4]{b^{48}}}}$

(g) $\sqrt{s\sqrt[3]{s\sqrt{s}}}$

(h) $\sqrt{x^2\sqrt{x}}$

(i) $\sqrt{\sqrt{a^{12}}}$

70. Explain the meaning of $\sqrt{1 + 2 + 3 + \dots}$.

71. Find the radicand: $\sqrt{\dots} = 36$.

72. $\sqrt{12 \text{ ft.}} = ?$

73. $4\sqrt{24} \text{ sq. ft.} = ?$

Study the set of numbers below.

$$\{5, 8, 11, 14, 17, \dots\} \quad (1)$$

Each term after the first can be obtained by adding a fixed number (in this case, 3) to the preceding term. Such a set of numbers is called an *arithmetic sequence*. Is the set below an arithmetic sequence?

$$\{1, 5, 9, 13, \dots\} \quad (2)$$

What is the common difference?

Review the work on the indicated pages.

74. In our studies, we have used the symbols

$$+, -, \times, \div. \quad (20-21)$$

We have also used the symbols

$$=, >, <. \quad (22-23)$$

Use each symbol in a mathematical statement.

ROMAN NUMERALS

Roman numerals are mathematical expressions and must be punctuated accordingly.

a. The English letter indicator must be used before a Roman numeral consisting of a single capitalized letter or before a Roman numeral consisting of one or more uncapitalized letters if the Roman numeral in braille is preceded by a space or by one or more punctuation marks and is followed by a space or by one or more punctuation marks.

(1) i, ii, iii, iv, v.

i. $4ab + 4x^2 - a^2 - 4b^2$

- (8) Solve problems i-v and ix-xi.

- (9) Read rule 3-i.

- (10) quadrant-I

- (11) Read chapters I-X and XV-XVI.

- (12) Solve Exercises I-a and II-b.

Figure 1 shows a 2x10 grid of dot patterns. The top row contains 10 patterns of 10 dots each, representing the digits 0-9. The bottom row contains 10 patterns of 10 dots each, representing the letters A-J. Each pattern is a 2x5 grid of dots.

- (13) Figure V—not VI—is greater in area.

- b. The English letter indicator must be used when it is necessary to show that a Roman numeral is printed in nonregular type.

- (1) Name the number shown by the boldface numerals: **I, ii.**

- c. The English letter indicator must not be used with a Roman numeral in regular type immediately preceded or followed by a sign of comparison. However, the English letter indicator must be used with a Roman numeral consisting of a single capitalized letter or one or more uncapitalized letters separated from a sign of comparison by a mark of punctuation.

- (1) $i = 1$, $v = 5$, and $x = 10$.

- (2) $I = 1$, $V = 5$, and $X = 10$.

- (3) $x_i = 10 + 1$

- (4) $LX = 50 + 10$

- (5) In Roman numerals, “C” = 100, “I” = 50, and “ix” = 9.

(2) (1, I, 2, II)



g. The English letter indicator must not be used with a Roman numeral in regular type enclosed in signs of grouping.

(1) 1. Add:

(i) $10,742 + 4,976$

(ii) $943 + 4632 + 1000$



(2) Locate quadrants (I) and (II).



h. When a Roman numeral is in direct contact with only its opening or closing sign of grouping, the English letter indicator must be used or must not be used as though the grouping signs were absent. However, the English letter indicator must not be used when a grouping sign carries a prime, a superscript, or a subscript.

(1) The problems (i and ii) are difficult.



(2) (M is a Roman numeral)



(3) (name the Arabic numeral for MCM)



(4) ($v = 5$)



(5) ($D = 500$)



(6) ($xiv = 14$)



(7) ($MCMLXXII = 1972$)



(8) ($v + i$) \times ($xx + xv$)



(9) $(V + I) \times (XX + XV)$



(10) Read sections X), X)', x)², and x)₂.



§92. Letter Combinations Similar to Roman Numerals: When it is unclear whether a letter combination is a Roman numeral, the combination must be treated as if it were not a Roman numeral. In such cases, the letters must be treated individually, and the English letter indicator must be used or not used in accordance with the rules for English letters.

(1) What does DC denote?



(2) dix has a special meaning.



SIGNS OF OPERATION (CONTINUED)

§93. Review of Signs of Operation: No space should be left before or after a sign of operation unless it is preceded or followed by a sign of comparison, an ellipsis, a dash, an unrelated word, or an abbreviation. However, no space must be left between an abbreviation and a fraction line. Signs of operation are mathematical symbols and must be punctuated accordingly.

The following signs of operation have already been introduced:

Plus	+	⠠⠭⠶
Minus	—	⠠⠤
Multiplication		
Cross (Cartesian product)	×	⠠⠭⠠⠭
Dot	•	⠠⠨
Division (divided by)	÷	⠠⠷⠠⠭
Plus or Minus	±	⠠⠫⠠⠭
Minus or Plus	∓	⠠⠫⠠⠤
Plus Followed by Minus	+ —	⠠⠭⠠⠤
Minus Followed by Plus	— +	⠠⠤⠠⠭

Minus Followed by Minus	--	
-------------------------	----	--

Fraction Lines (over, divided by)

Horizontal Simple Fraction Line	—	
Diagonal Simple Fraction Line	/	
Horizontal Complex Fraction Line	—	
Diagonal Complex Fraction Line	/	

§94. Additional Signs of Operation:

a. Ampersand (and, logical conjunction): &

When the ampersand is used in mathematical context, it must be treated as a sign of operation, and the symbol shown above must be used. However, when the ampersand is used in abbreviations or other literary context, the symbol and rules of English braille apply.

(1) A & B

(2) The & denotes *logical conjunction*.

(3) AT&T

(4) The B & O Railroad.

b. Asterisk

*

Crosshatch—number sign,
tic-tac-toe, pounds (weight)

#

Paragraph Mark

¶

Section Mark

§

The asterisk, crosshatch, paragraph, and section marks must be represented by the symbols of Nemeth code; English braille symbols must not be used. If these signs of operation occur as superscripts or subscripts, their position must be shown.

The numeric indicator must be used before a numeral or a decimal point and a numeral following an asterisk, crosshatch, paragraph, or section mark.

(1) $f * g$



(2) $(1 + 2) * (3 + 4)$



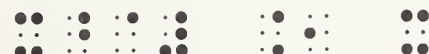
(3) $1 * 2$



(4) $.1 * .2$



(5) $x^* > x$



(6) $2\pi x_k^* \cdot \Delta x_k \cdot 2y_k^*$



(7) A'^*



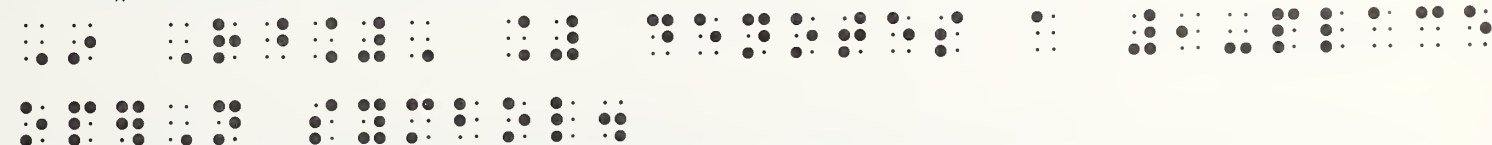
(8) $x \# y = y \# x$



(9) $1 \# 2 = 2 \# 1$



(10) In $R^\#$, $\#$ denotes a 1-place operation symbol.



(11) $A \P B$



(12) $3 \P 4 = 4 \P 3$



(13) $A \S B$



(14) $3 \S 4 = 4 \S 3$



c. Back Slash (divides, is a factor of) :



(1) $b \backslash a$ can be read as "b divides a."



(2) $3 \backslash 6$ denotes "3 is a factor of 6."



d. Dagger:

Single

†



Double

‡



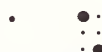
(1) $A \dagger B = B \dagger A$



(2) $A \ddagger B = B \ddagger A$



e. Dot (and, times):



The dot may be used as a multiplication sign or to denote "and."

(1) In logic, $p \cdot q$ is read "p and q."



f. Hollow Dot: °



The hollow dot may be used as a sign of operation or as a superscript to represent degrees of temperature or angle.

(1) $f \circ g$



(2) $a \circ (b \circ c) = (a \circ b) \circ c$



(3) $f_1 \circ f_2 \circ \dots \circ f_n$



(4) $40^\circ + 50^\circ = 90^\circ$



(5) $\frac{1}{2}^\circ + 1\frac{1}{2}^\circ = ?^\circ$



(6) $28^\circ 6' 37''$



(7) A 30° - 60° - 90° triangle.



(8) θ_2°



(9) $100^\circ \text{ C.} = 212^\circ \text{ F.}$



(10) $45^\circ \text{ C} + 5^\circ \text{ C} = 50^\circ \text{ C}$



(11) N. 35° W.



(12) $(60^\circ, 70^\circ, 80^\circ)$



(13) $45^\circ, 90^\circ.$



g. Intersection (cap): \cap



(1) $A \cap B = B \cap A$



(2) $(A \cap B) \cap C = A \cap (B \cap C)$



(3) Prove $\cap_i A_i = U$.



h. Logical Product (and, meet): \wedge



(1) $p \wedge q \wedge r$



(2) $(p_1 \wedge p_2 \wedge p_3 \wedge \dots \wedge p_n)$



i. Logical Sum (or, join): \vee

(1) $p \vee q \vee r$



(2) $[p \vee (q \vee r)]$



j. Minus With Dot Over (proper difference): $\dot{-}$

(1) $x \dot{-} y = 0$



(2) $x \dot{-} y' = \text{pd}(x \dot{-} y)$



k. Tilde (not):

Simple \sim

Extended \sim

In logic, the tilde is used as a sign of operation meaning "not."

When two symbols for the tilde follow one another, the multipurpose indicator must be inserted between them to indicate that they are written horizontally.

(1) $\sim p$



(2) $\sim p \vee q$



(3) $\sim p \wedge \sim q \wedge \sim r$



(4) $\sim (\sim p)$



(5) $\sim \sim p \vee q$



(6) $\sim s \vee t$



l. Union (cup): \cup



(1) $A \cup B = B \cup A$



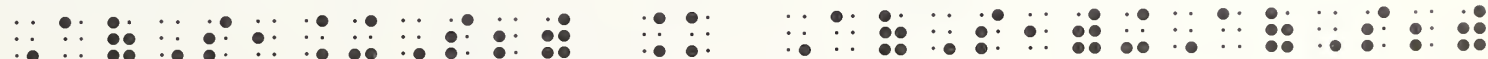
(2) $(A \cup B) \cup C = A \cup (B \cup C)$



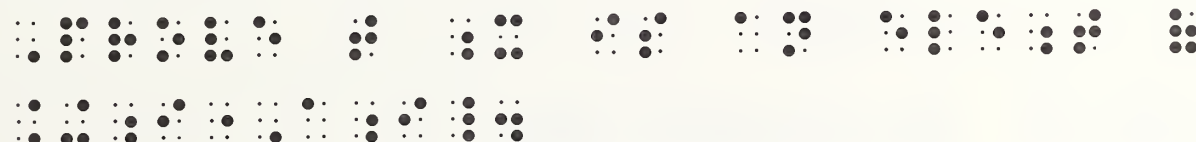
(3) $\{a, b, c, d\} \cup \{c, d\}$



(4) $A(S_1 \cup S_2) = A(S_1) + A(S_2)$



(5) Prove that x is an element of $\cup_i A_i$.



m. Vertical Bar (is a factor, divides): $|$



The vertical bar is used both as a sign of grouping and as a sign of operation.

(1) In $b | a$, b is a factor of a .



(2) $6 | 12$ can be read as "6 divides 12."



(3) $x + 2 | x^2 + 7x + 10$



n. Vertical Bar Negated (does not divide): \nmid





(1) $5 \nmid n$



§95. Signs of Operation and Boldface Type: The signs of operation listed below are to be used to show boldface type only when the distinction between the regular and the boldface forms of the same sign has mathematical significance. Each symbol consists of dots

4-5-6 followed by the appropriate sign of operation. In this case, dots 4-5-6 must be considered not as the boldface type-form indicator but as part of its related symbol. Dots 4-5-6 must not be used with any other sign of operation.








Boldface Plus	+	
Boldface Minus	—	
Boldface Plus Followed by Boldface Minus	+ —	
Boldface Plus Followed by Regular Minus	+ —	
Regular Plus Followed by Boldface Minus	+ —	
Boldface Minus Followed by Boldface Plus	— +	
Boldface Minus Followed by Regular Plus	— +	
Regular Minus Followed by Boldface Plus	— +	

- (1) $a + b = b + a$
- 
- (2) $a + - c = b - + d$
- 

REFERENCE SYMBOLS

§96. **Reference Symbols and Punctuation:** The asterisk, dagger, paragraph mark, section mark, and star must be represented by the symbols listed below; English braille symbols must not be used. Some of these signs are also used as signs of operation. However, when used as reference symbols, they must be transcribed in accordance with the following rules.

A reference symbol or numeral printed in a superscript position must not be shown as a superscript in braille. Reference symbols must be punctuated mathematically.

Asterisk	*	
Dagger	†	
Double	‡	
Paragraph Mark	¶	
Section Mark		
Single	§	
Double	§§	
Star	☆	

§97. References to Footnotes:

a. Reference Symbols Denoting Footnotes: When a printed symbol referring to a footnote is attached to the beginning or end of a word, mathematical expression, etc., the reference symbol must follow that item with a space between in braille and must appear on the same braille line. When such a reference symbol is unattached, it must be positioned as it appears in print. No space should be left between a footnote reference symbol and its related punctuation mark.

- (1) Find the *quarterly interest.

- (2) Find the quarterly* interest.



- (3) Let R^\dagger be a set.

- (4) Let R be a set. †

The figure shows a sequence of 12 diagrams, each representing a 3x3 grid of dots. The sequence starts with a single dot in the center (Diagram 1) and progresses through various patterns of dots, including horizontal and vertical lines, and eventually fills the grid. The diagrams are arranged in a single row, separated by arrows indicating the sequence.

- (5) Find the quotient*.

- (6) The asterisk (*) denotes a footnote.

b. Numbered or Lettered Footnotes and the General Reference Indicator:   When a footnote reference is denoted by a numeral alone, the general reference indicator and the numeric indicator must be placed before the note numeral. When a footnote reference is denoted by a letter alone, the general reference indicator must be placed before the note letter. The English letter indicator must not be used.

An attached footnote reference numeral or letter must follow the corresponding item. An unattached footnote reference numeral or letter must be positioned as it appears in print. Except for its punctuation, the general reference indicator and its numeral or letter must be preceded and followed by a space.

- (1) Find the interest¹ and annuity².

- (2) In $\sqrt[3]{a}$, the index of the radical is 3.

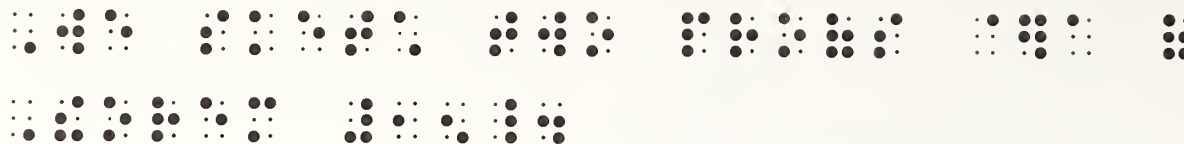
Figure 1 displays two styles of handwritten digit representations (0-9) on a 3x3 grid. The top row shows 'Style 1' and the bottom row shows 'Style 2'. Each digit is represented by a 3x3 grid of dots, with some dots filled (black) and others empty (white).

Digit	Style 1 (Top Row)	Style 2 (Bottom Row)
0	Top-left, top-middle, top-right, middle-left, middle-right, bottom-left, bottom-middle, bottom-right	Top-left, top-middle, top-right, middle-left, middle-right, bottom-left, bottom-middle, bottom-right
1	Top-middle, middle-right, bottom-right	Top-middle, middle-right, bottom-right
2	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left
3	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left
4	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left
5	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left
6	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left
7	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left
8	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left
9	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left	Top-left, top-middle, top-right, middle-right, bottom-right, bottom-middle, bottom-left

- (3) If B is a formula and y is a variable, ¹ then $(y)B$ is a formula.

The figure displays two rows of 12 dot patterns each, representing the digits 0 through 9. The top row shows the digits in a standard 7-segment display style, while the bottom row shows the digits in a more complex, stylized dot pattern style. The digits are arranged in two rows of six.

- (4) We sketch two proofs^a of Theorem 15.



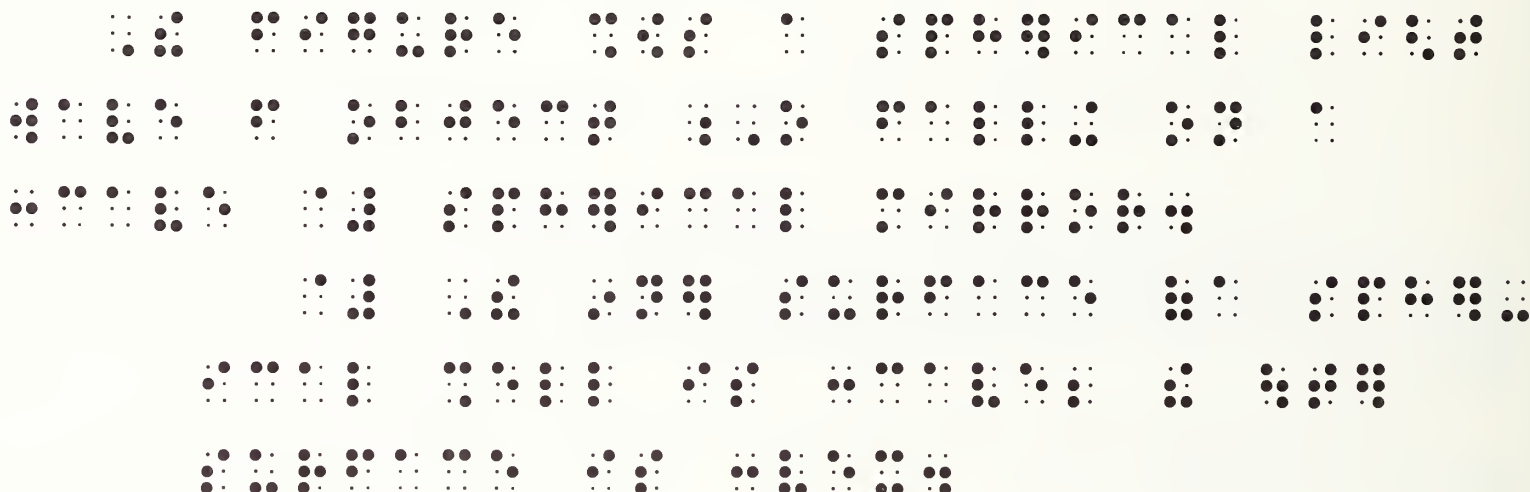
- (5) Other external problems will be found in the exercises.^b



c. **Placement of Footnotes:** In transcribing footnotes, the reference symbol and its related footnote must be positioned and indented in accordance with the rules of the *Code of Braille Textbook Formats and Techniques*.

- (1) The figure shows a spherical light wave from object O falling on a concave* spherical mirror.

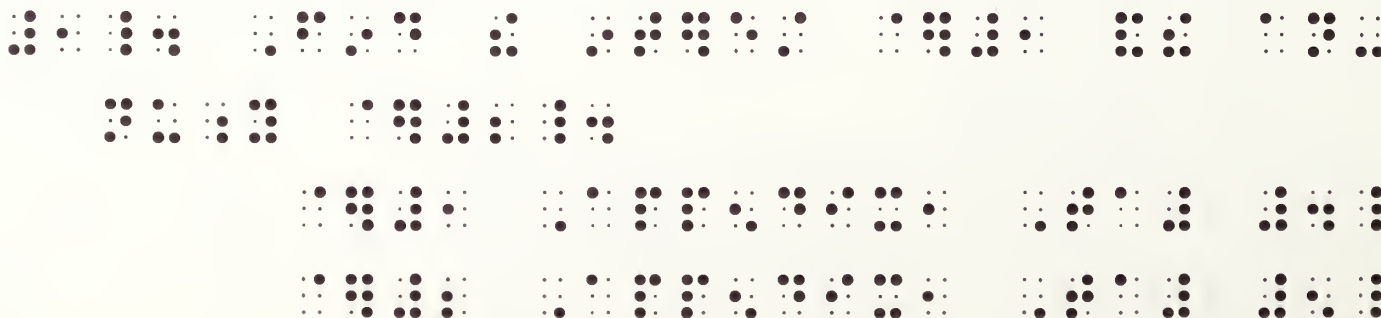
* The inner surface of a spherical shell is concave; the outer surface is convex.



- (2) 1. Find the interest¹ and the annuity².

¹Appendix, Table 4.

²Appendix, Table 5.



§98. References to Items Other Than Footnotes:

a. A reference symbol may be used with lettered or numbered items to indicate work for extra practice or credit, or to denote references to specific sections, paragraphs, etc. Such a reference symbol must be positioned as it appears in print and must be unspaced from its related letter or numeral. The numeric indicator must be used before a numeral following a reference symbol.

- (1) *1. Find $(-1)^{10}$.

- (2) 1*. What is 5^7 ?

- (3) 1.* Factor: $6x^2 - 5x$.

- (4) Read §12, ¶a.

- (5) Study ¶1, §i and §ii.

- (6) Prove Theorem 1 (§A, ¶4).

b. When a reference symbol is used to single out one or more words for special emphasis, a space must be left between the reference symbol and the word.

- (1) ☆ complex fraction

§99. Unlisted Reference Symbols: When print symbols other than those listed are used for reference, the equivalent braille symbols in the Nemeth code must be used. If no equivalent braille symbol exists, a symbol must be devised and explained in a transcriber's note. All such reference symbols must be transcribed in accordance with the rules stated above.

HOMEWORK

Prepare the following homework for submission to your teacher. Proofread carefully.

EXERCISE 8

1. The capitalized letters I, V, X, L, C, D, and M are the number symbols we use to write Roman numerals. What Arabic number does each Roman numeral name?
2. Use Fig. 3 to name the ordered pairs in quadrants I, II, III, and IV.

3. A cornerstone is marked MCMLXXII. What date does it represent?
4. Write in Arabic numerals:
 - a. VI
 - b. XIII
 - c. MCXL
5. The uncapitalized letters i, v, x, l, c, d, and m are also used to write Roman numerals. What number does each represent?
6. What number does each of the following name: i, ii, iv, vii, x.
7. Solve $V = lwh$ for each of the following (use Figure 3.4):
 - i. for l
 - ii. for w
 - iii. for h
8. List the definitions given in Chapters I-V and XI-XV.
9. Do Exercises I-a and IV-c.
10. Refer to Theorem 5.9 and show that T has the properties i-ii and iv-v.
11. True or false:

a. $I = 1$	b. $VII = V + II$
c. $XLI - IV = XXXVII$	d. $i = 1$
e. $x + ii = 10 + 2$	f. $xv - ix = xiv$
12. Does “V” = 5? Are V and 5 names for the same number?
13. If “x” denotes 10, does “x” = “10”?
14. Refer to Formulas I and I'. Apply the principles to Theorems I and II².
15. Read items i, i', and i₂.
16. Are the I's added or subtracted in II, IV, VI? What value do the XX's have in LXX?
17. In sets A, B, and C where

$$A = \{1, II, 3, IV, 5\}$$

$$B = \{i, ii, iii, iv, v\}$$

$$C = \{I, 2, III, 4, V\}$$
 is there a 1-to-1 correspondence between the members of sets A and B? between A and C? between B and C?
18. Write the proofs for (i), (ii), and (iii) of Theorem 3.2. Use the results to explain figures (I) and (II).
19. Use the figures (I and II) to solve the problems (i and ii).
20. Read sections I), I)², and II).
21. The symbol & is used to show conjunction. Translate P&C into a word sentence.
22. If 1, 2, and 3 are elements of a system in which two operations are denoted by the symbol # and the symbol &, which of the following expressions represents the statement # is distributive over &?
 - (a) $1 \# (2 \& 3) = (1 \& 2) \# (1 \& 3)$
 - (b) $1 \# (2 \& 3) = (1 \# 2) \& (1 \# 3)$
 - (c) $1 \& (2 \# 3) = (1 \# 2) \& (1 \# 3)$
23. Explain how the cardinal number of a set M is the same as the cardinal number of a set N, or $\#(M) = \#(N)$.

24. An operation $(*)$ has been defined over the set of natural numbers. In each of the following, find $2 * 3$.
- $x * y = (1 + x) + y$
 - $x * y = x(2y)$
 - $x * y = xy^2$
25. Prove: If f has the property that, for all a, b , and c in X , then
- $$a * (b * c) = (a * b) * c.$$
26. Prove that for any order types α and β , $(\alpha + \beta)^* = \alpha^* + \beta^*$.
27. Find the volume of the k th shell where (x_k^*, y_k^*) is the point of the abscissa $x_k^* = \frac{1}{2}(x_k + x_{k+1})$.
28. Find the conditional density of Y_1^* , given Y_2^* .
29. A binary operation \P has the cancellation property if and only if each of $x \P z = y \P z$ and $z \P x = z \P y$ implies $x = y$. Show how this applies to the equality relation. Does $1 \P 3 = 4 \P 3$?
30. Y is a set, \S is an associative operation in Y , and m is a member of Y such that $m \S y = y \S m = y$ for all y in Y . Use this notation for all operations in Y .
31. If $p \dagger q$ is defined as "neither p nor q ," show that $[(p \dagger p) \dagger (q \dagger q)]$, if and only if $(p \cdot q)$ is a tautology.
32. Prove: For all integers a, b , and c , if a and b are relatively prime and $b \backslash ac$ then $b \backslash c$. Is this true for $2 \backslash 8 \times 4$?
33. Given: a, b, c , and x denote elements of the set of natural numbers. Prove:
- If $b \mid a$ and $a \mid c$, then $b \mid c$.
 - If $b \mid a$ and $b \mid c$, then $b \mid (a + c)$.
 - If p is a prime and $p \mid a^2$, then $p \mid a$.
34. From the given information, show that if $a = h$ and $b = k$, then $a \circ b = h \circ b$, $a \circ b = a \circ k$, and $a \circ b = h \circ k$.
35. State whether each equation is valid for linear functions f, g , and h .
- $(f + g) \circ h = f \circ h + g \circ h$
 - $f \circ (g \circ h) = (f \circ g) \circ (f \circ h)$
36. Prove: If $90^\circ = 90^\circ$ and $x^\circ > y^\circ$, then $(90 - x)^\circ < (90 - y)^\circ$.
37. Find the answer:
- $68^\circ - 32^\circ = ?^\circ$
 - $\frac{3}{4}^\circ + 1\frac{1}{4}^\circ = ?^\circ$
38. Find the sum of $30^\circ 28'$ and $56^\circ 42'$.
39. A 30° - 60° - 90° -triangle is what kind of triangle? Draw a 45° - 45° - 90° -triangle.
40. Water freezes at 32° F (32 degrees Fahrenheit). Is this the same as 0° C (0 degrees Centigrade)?
41. Let sets $S = \{0, 1, 2, 3\}$ and $T = \{5, 6, 7, 8, 9\}$. Find
- $S \cup T$
 - $T \cup S$
 - $S \cap T$
 - $T \cap S$
42. What laws are illustrated by
- $$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \text{ and}$$
- $$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)?$$

43. Using Venn diagrams show that $(A \cup B)^c = A^c \cap B^c$.
44. We express the union of three sets as $A_1 \cup A_2 \cup A_3$ and, more generally, of n sets as $A_1 \cup A_2 \cup \dots \cup A_n$.
45. Given: Statements p and q are true and r is false. Identify each of the following as true or false.
- (a) $p \wedge q$ (b) $p \wedge (q \vee r)$ (c) $(p \wedge q) \vee r$
 (d) $(\sim p \wedge q)$ (e) $\sim p \wedge \sim q \wedge \sim r$
46. True or false: Any proposition is the negation of its own negation, or $p = \sim(\sim p)$.
47. Given: Alice is at home and Tom is at home. Which statements are true:
- (a) $p \cdot q$ (b) $\sim \sim(p \cdot q)$
48. We define the proper difference, $\dot{-}$, by the notation $m \dot{-} 0 = m$ and $m \dot{-} n' = \text{pd}(m \dot{-} n)$. That is, $m \dot{-} n = m - n$ if m and n are not greater than or equal to each other, and $m \dot{-} n = 0$ if $m < n$. Obtain $m \dot{-} n$ as the value of $\dot{-}$ at $\langle m, n \rangle$.
49. Special operations are denoted by a boldface plus ($\mathbf{+}$) and a boldface minus ($\mathbf{-}$). Use induction to determine if $a \mathbf{+} b = b \mathbf{+} a$ and $c \mathbf{-} d = d \mathbf{-} c$.
50. Show $a/b = c/d$ iff* $ad = bc$.
51. The diagram shows two fixed points A and B and a ray APB^\dagger connecting them.
52. In $\sqrt[n]{a}$, n is the index¹ of the radical. What is a ?

*The phrase "if and only if" occurs so often in mathematics that the abbreviated form "iff" is used throughout this text.

[†]Assume that ray APB lies in the plane of the figure.

¹If no index is indicated, it is understood to be 2.

SYMBOLIC LOGIC

Mathematical or symbolic logic is the systematic study of the basic principles of valid relationships by a method which makes a distinction between the validity of some reasoning and the truth of the premises from which it was derived.

In our work we will be using the words and phrases listed below. Study the designated pages and give examples for each starred item. The exercise following the list is for extra study. Such study is denoted by the use of an asterisk placed before the problem number.

☆ composite proposition (31)

☆ conjunction (32)

☆ disjunction (33)

☆ complete disjunction (34)

*53. Show that definition 3.4, §a, ¶1, is a tautology.

LESSON 9

REPRESENTATION OF ARROWS

§100. **General Principles:** A figure in the form of an arrow is represented by braille symbols suggesting the ink-print shape. In representing arrows, the appropriate arrowhead and arrow shaft must be selected from the following list.

Arrowheads

Left Full Barbed	<	
Left Lower Barbed	↙	
Left Upper Barbed	↖	
Right Full Barbed	>	
Right Lower Barbed	↘	
Right Upper Barbed	↗	
Left Full Blunted	└	
Left Lower Blunted	└	
Left Upper Blunted	└	
Right Full Blunted	┘	
Right Lower Blunted	┘	
Right Upper Blunted	┘	
Left Full Curved	⤵	
Left Lower Curved	⤵	
Left Upper Curved	⤵	
Right Full Curved	⤴	
Right Lower Curved	⤴	
Right Upper Curved	⤴	
Left Full Straight	┆	

Left Lower Straight	└	
Left Upper Straight	┐	
Right Full Straight	├	
Right Lower Straight	┘	
Right Upper Straight	┙	

Arrow Shafts

Ordinary Length Single	—	
Ordinary Length Double	==	
Short Single	—	
Short Double	==	
Long Single	—	
Long Double	==	
Curved	(or)	
Dashed	--	
Dotted	...	
Wavy	~	

§101. **Construction of Arrows:** In general, arrows should be transcribed in the order in which their symbols appear.

a. **Shape Indicator:**























The shape indicator must be used before an arrow symbol.



b. **Arrowheads:** An arrowhead may be barbed, blunted, straight, or curved, and may occur at the left, at the right, or at both ends of an arrow shaft. An arrowhead may also appear with only its upper or lower portion.





















c. **Arrow Shafts:** The length of an arrow shaft is indicated by the number of times the braille arrow shaft symbol is used. One braille symbol represents a short shaft, two symbols represent the ordinary shaft length, and three or more symbols are used to indicate a longer shaft.

- (1) →
- (2) ←

(3)	\longleftrightarrow	
(4)	\longrightarrow	
(5)	\longleftarrow	
(6)	\longleftrightarrow	
(7)	\longrightarrow	
(8)	\longleftarrow	
(9)	\longleftrightarrow	
(10)	\Rightarrow	
(11)	\Leftarrow	
(12)	\longleftrightarrow	
(13)	\longrightarrow	
(14)	$\cdots>$	
(15)	$\langle\rangle$	
(16)	\lceil	
(17)	\rfloor	
(18)	\lceil	
(19)	\rceil	
(20)	\lceil	
(21)	\lceil	
(22)	\lceil	
(23)	\lceil	
(24)	\lceil	
(25)	\lceil	

(26)		
(27)		
(28)		
(29)		
(30)		
(31)		
(32)		
(33)		
(34)		
(35)		
(36)		

d. **Arrows With Dotted Ends:** Arrows may also be represented by a shaft preceded or followed by a solid dot (•)  or by a hollow dot (◦)  .

(1)		
(2)		
(3)		
(4)		
(5)		
(6)		
(7)		
(8)		
(9)		
(10)		

§102. **Arrow Directions:** Arrow directions are represented by the use or nonuse of direction indicators. When required, the appropriate direction indicator must immediately follow the shape indicator.

a. Horizontal Arrow Directions: Direction indicators are not required with a horizontal arrow which points left, right, or both left and right.

- (1) ← ⠠⠠⠠⠠
- (2) → ⠠⠠⠠
- (3) ⇒ ⠠⠠⠠⠠
- (4) ⇔ ⠠⠠⠠⠠⠠

b. Vertical Arrow Directions: Vertical arrows require the following direction indicators:

- Directly-Over Indicator** ⠠
- Directly-Under Indicator** ⠡

The directly-over indicator must be used before a two-headed vertical arrow pointing up and down, or before a one-headed vertical arrow pointing up only.

The directly-under indicator must be used before a one-headed vertical arrow pointing down.

When a vertical arrow has only one arrowhead, the symbol for the right-arrowhead must be used.

- (1) ⇕ ⠠⠠⠠⠠⠠
- (2) ↑ ⠠⠠⠠⠠
- (3) ↓ ⠠⠠⠠⠠

c. Slanted Arrow Directions: Slanted arrows require the following direction indicators:

- Superscript Indicator** ⠠
- Subscript Indicator** ⠡





The superscript indicator must be used to show that a one-headed left- or right-slanted arrow points upward. The subscript indicator must be used to show that a one-headed left- or right-slanted arrow points downward.


- (1) ↖ ⠠⠠⠠⠠
- (2) ↗ ⠠⠠⠠⠠
- (3) ↙ ⠠⠠⠠⠠
- (4) ↘ ⠠⠠⠠⠠













When a slanted arrow has two arrowheads, the superscript indicator is used when the left arrowhead points upward; the subscript indicator is used when the left arrowhead points downward.

- (5)  
- (6)  




d. Curved Arrow Directions: Direction indicators are not required with curved arrows. The direction of curvature is shown by a left- or right-arrowhead. A curved shaft followed by a right-arrowhead represents a counterclockwise arrow; a curved shaft preceded by a left-arrowhead represents a clockwise arrow.

- (1)  
- (2)  

§103. Boldface Arrows: When an arrow is printed in boldface type, the boldface type-form indicator  must be placed before the arrowhead or arrow shaft at the beginning of the arrow symbol. When both the boldface type-form indicator and a direction indicator are required, the boldface type-form indicator must follow the direction indicator.

- (1)  
- (2)  
- (3)  
- (4)  
- (5)  
- (6)  

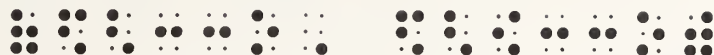
§104. Spacing and Punctuation With Arrows: Arrows are considered signs of comparison and must be spaced and punctuated accordingly.

- (1) $B \leftarrow A$

- (2) $p \longleftrightarrow q$

- (3) $|x - a| < 3 \longleftrightarrow x - a$


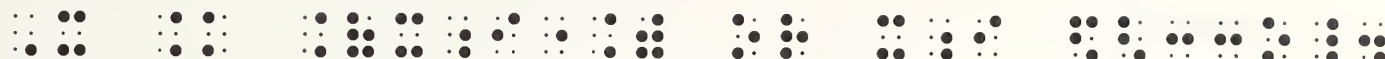
(4) $x \downarrow 2$



(5) (\uparrow, \downarrow)



(6) $X = \{x_i\} \text{ or } x_i \uparrow$



§105. Contracted Form of Right-Pointing Arrow: \longrightarrow

The contracted form for a right-pointing arrow must be used when a right-pointing arrow with a full barb and a single shaft of ordinary length occurs by itself in regular type. All other right-pointing arrows always require the use of all appropriate symbols.

(1) $A \longrightarrow B$



(2) $n:v \longrightarrow r$



(3) $2\text{H}_2\text{O} + \text{O}_2 \longrightarrow 2\text{H}_2\text{O}$



(4) $p \longrightarrow (q \vee r)$



(5) $X \longrightarrow Y$



(6) $x \longrightarrow y$



(7) $X \Longrightarrow Y$



(8) $X \longrightarrow Y$



SIGNS OF COMPARISON (CONTINUED)

§106. **Review of Signs of Comparison:** A space must be left before and after a sign of comparison. However, no space should be left between a sign of comparison and a sign of grouping, a braille indicator, or a punctuation mark which is related to it.

The following simple signs of comparison have already been introduced:

Arrow

Pointing Left	←	⠠⠠⠠⠠
Pointing Right	→	⠠⠠⠠⠠
Contracted Form	→	⠠⠠
Uncontracted Form	→	⠠⠠⠠⠠
Pointing Left and Right	↔	⠠⠠⠠⠠⠠⠠
Pointing Up	↑	⠠⠠⠠⠠⠠⠠
Pointing Down	↓	⠠⠠⠠⠠⠠⠠
Pointing Up and Down	↕	⠠⠠⠠⠠⠠⠠⠠⠠
Equals	=	⠠⠠
Greater Than (is greater than)		
With Straight Sides	>	⠠⠠
With Curved Sides	>	⠠⠠⠠
Less Than (is less than)		
With Straight Sides	<	⠠⠠
With Curved Sides	<	⠠⠠⠠
Proportion (as)	::	⠠⠠
Ratio (is to)	:	⠠⠠

§107. Additional Simple Signs of Comparison:

a. Arc:

Concave Upward	⌒	⠠⠠
Concave Downward	⌓	⠠⠠

(1) $x \frown y$



(2) $x \smile y$



b. Equivalence (is equivalent to): \approx

(1) $x \approx y$



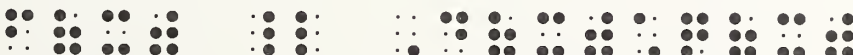
c. Identity (is identical with, is congruent to): \equiv

This symbol must not be used for *is congruent to* if another sign is used for this purpose in print.

(1) $A + B \equiv B + A$



(2) $f(x) \equiv D(x) \cdot q(x)$



d. Inclusion (is contained in, is a subset of): \subset

(1) $A \subset D$



(2) $(A \cup E) \subset (F \cup B)$



e. Membership (is an element of, belongs to): ϵ or \in or \in

This symbol is generally used for sets and their elements. It must not be mistaken for the Greek uncapitalized epsilon even though it may be referred to as such.

(1) $5 \in B$



(2) $\frac{8 \times 4}{4} \in \{8\}$



f. **Parallel To (is parallel to):** \parallel

(1) $AB \parallel CD$

g. **Perpendicular To (is perpendicular to):** \perp

(1) $PQ \perp RS$

(2) $x = s + s^\perp$

h. **Relation (is related to):** R

When an R or any other letter or sign is used between two expressions to show relation, the letter or other symbol must be treated as a sign of comparison.

(1) $a R b$

(2) $r \theta s$

i. **Reverse Inclusion (contains; in logic, implies):** \supset

(1) $D \supset A$

(2) $C \supset C_1 \supset C_2 \supset \dots$

j. **Reverse Membership (contains the element):** \ni or \exists or \ni

(1) $B \ni 5$

k. **Tilde:**

Simple (is related to, is similar to) \sim

Extended (is related to) \sim

The tilde may be used as a sign of operation or as a sign of comparison.

(1) $PQR \sim P'Q'R'$



(2) $x \sim y$



l. Variation (varies as): \propto

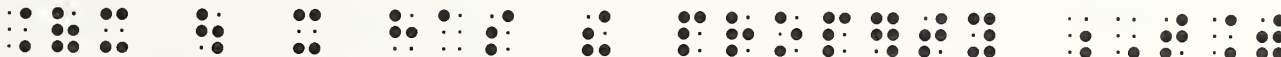
(1) $x \propto y$



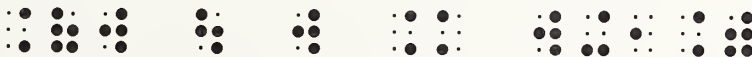
m. Vertical Bar (such that): $|$

The vertical bar may be used as a sign of grouping, as a sign of operation, or as a sign of comparison. When used as a sign of comparison, it usually occurs in an expression within braces used for set notation. However, it may also appear in other situations.

(1) $\{x \mid x \text{ has the property } T\}$



(2) $\{w \mid w = w + 1\}$



(3) $\{m \mid 3(m - 6) = -9\}$



(4) $\{(x, y) \mid x + y < 6\}$



§108. Signs of Comparison Compounded Vertically: When two or more simple signs of comparison are arranged one under the other, the combination becomes a single comparison sign compounded vertically. The symbol for the uppermost sign must be written first and unspaced from the symbol for the next lower sign.

a. **Arrows:** When a right-pointing arrow in regular type with a full barb and single shaft of ordinary length is part of a sign of comparison compounded vertically, its contracted form must not be used.

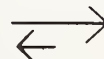
Pointing-Right Over Pointing-Left



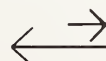
Pointing-Left Over Pointing-Right



Long Pointing-Right Over Short Pointing-Left



Short Pointing-Right Over Long Pointing-Left



Pointing-Right With Upper Barb Only Over
Pointing-Left With Lower Barb Only



Pointing-Right Over Boldface Pointing-Left



Pointing-Left Over Boldface Pointing-Right



Boldface Pointing-Right Over Pointing-Left



Boldface Pointing-Left Over Pointing-Right



Boldface Pointing-Right Over Boldface Pointing-Left



Boldface Pointing-Left Over Boldface Pointing-Right




(1) $A + B \rightleftharpoons C + D$



(2) $HCl \rightleftharpoons H^+ + Cl^-$



b. Equals:

Horizontal Bar — 

Oblique Stroke / or \ 

Equals Sign = 

As shown in c through k below, a single horizontal bar, an oblique stroke, or an equals sign may appear as part of a sign of comparison compounded vertically. The horizontal bar or oblique stroke is often substituted for the equals sign.

c. Greater Than:

Bar Over Greater Than (is equal to or greater than)

$\overline{>}$ or \geq



Equals Sign Over Greater Than (is equal to or greater than)

$\overline{=}$ or \geq



Bar Under Greater Than (is greater than or equal to)

$\underline{\geq}$ or \geq



Equals Sign Under Greater Than (is greater than or equal to)

$\underline{=}$ or \geq



(1) $a \geq b$



(2) $a \overline{>} b$

(3) $|x| \geq 0$

(4) $|x| \geq 0$

d. Inclusion (is a subset of) :

Bar Over Inclusion $\overline{\subset}$

Equals Sign Over Inclusion $\overline{=}$

Bar Under Inclusion $\underline{\subset}$

Equals Sign Under Inclusion $\underline{=}$

(1) $C \overline{\subset} B'$

(2) $C \overline{= \subset} B'$

(3) $(D \cap E) \subseteq (E \times E)$

(4) $(D \cap E) \underline{= \subseteq} (E \times E)$

e. **Intersection (cap):** The intersection sign is a sign of comparison when modified by a bar or equals sign above or below it. An unmodified intersection sign is a sign of operation (see §94g).

Bar Under Intersection $\underline{\cap}$

Equals Sign Under Intersection $\underline{=}$

(1) $X \underline{\cap} Y$

(2) $X \underline{= \cap} Y$

f. Less Than:

Bar Over Less Than (is equal to or less than)

$\overline{<}$ or \leq



Equals Sign Over Less Than (is equal to or less than)

$\overline{=}$ or \leq



Bar Under Less Than (is less than or equal to)

\leq or \leq



Equals Sign Under Less Than (is less than or equal to)

\leq or \leq



(1) $v - 1 \overline{<} 5$



(2) $v - 1 \overline{=}< 5$



(3) $6 \leq x \leq 9$



(4) $6 \leq x \leq 9$



g. **Logical Product (meet):** The logical product sign is a sign of comparison when modified by a bar or equals sign above or below it. An unmodified logical product sign is a sign of operation (see §94h).

Bar Over Logical Product

$\overline{\wedge}$



Bar Over and Bar Under Logical Product

$\overline{\wedge}$



Bar Over and Equals Sign Under Logical Product

$\overline{\wedge}$



Bar Under Logical Product

\wedge



Equals Sign Over Logical Product

$\overline{\wedge}$



Equals Sign Over and Bar Under Logical Product

$\overline{\wedge}$



Equals Sign Over and Equals Sign Under Logical Product

$\overline{\wedge}$

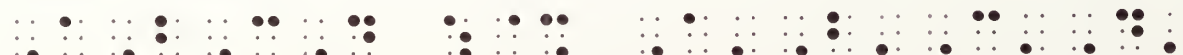


Equals Sign Under Logical Product

\wedge



(1) $ABCD \overline{\wedge} A'B'C'D'$



(2) $\{A\} \underline{\underline{\wedge}} K$



(3) $p \underline{\wedge} q$



h. **Logical Sum (join):** The logical sum sign is a sign of comparison when modified by a bar or equals sign above or below it. An unmodified logical sum sign is a sign of operation (see §94i).

Bar Over Logical Sum



Bar Over and Bar Under Logical Sum



Bar Over and Equals Sign Under Logical Sum



Bar Under Logical Sum



Equals Sign Over Logical Sum



Equals Sign Over and Bar Under Logical Sum



Equals Sign Over and Equals Sign Under Logical Sum



Equals Sign Under Logical Sum



(1) $ABC \overline{\vee} A'B'C'$



(2) $F_1 \overline{\overline{\vee}} F_2$



(3) $P(E \underline{\vee} F)$



i. **Reverse Inclusion:**

Bar Over Reverse Inclusion



Equals Sign Over Reverse Inclusion



Bar Under Reverse Inclusion



Equals Sign Under Reverse Inclusion



(1) $B \supset A$



(2) $D \supset C$



j. Tilde:

Bar Over Single Tilde

12



Equals Sign Over Single Tilde

112



Double Tilde (is approximately equal to)

22



Bar Over Double Tilde

122



Equals Sign Over Double Tilde

1122



Bar Under Single Tilde

21



Equals Sign Under Single Tilde (is congruent to)

211



Bar Under Double Tilde

221



Equals Sign Under Double Tilde

2211



(1) $3.14159 \approx 3.1416$



(2) $ABC \cong DEF$



k. Union (cup): The union sign is a sign of comparison when modified by a bar or equals sign above or below it. An unmodified union sign is a sign of operation (see §94I).

Bar Under Union

U



Equals Sign Under Union

U



(1) $A \cup B$



(2) $X \cup Y$



§109. Signs of Comparison Compounded Horizontally: When two or more signs of comparison are arranged side by side, the combination becomes a single comparison sign compounded horizontally. The multipurpose indicator $\cdot\cdot$ must be placed between the unspaced symbols to indicate that they are printed horizontally, not vertically. Unlisted signs of comparison compounded horizontally must be transcribed according to the same principle.

a. Arrows:

Pointing-Up Followed by Pointing-Down



Pointing-Down Followed by Pointing-Up



Pointing-Up Followed by Boldface Pointing-Down



Pointing-Down Followed by Boldface Pointing-Up



Boldface Pointing-Up Followed by Pointing-Down



Boldface Pointing-Down Followed by Pointing-Up



Boldface Pointing-Up Followed by Boldface Pointing-Down



Boldface Pointing-Down Followed by Boldface Pointing-Up



(1) $+ H_2O \updownarrow - H_2O$



b. Greater Than:

Followed by Less Than

$> <$



Followed by Equals Sign Followed by Less Than

$> = <$



(1) $n > < 1$



(2) $n > = < 1$



c. Less Than:

Followed by Greater Than

$< >$



Followed by Equals Sign Followed by Greater Than

$< = >$



HOMEWORK

Prepare the following homework for submission to your teacher. Proofread carefully.

EXERCISE 9

1. Find functions f and g and a number n such that $f(x) \rightarrow 0$ and $g(x) \rightarrow 0$ as $x \rightarrow n$ and such that
 - a. $f(x)/g(x) \rightarrow 1$ as $x \rightarrow n$
 - b. $f(x)/g(x) \rightarrow 0$ as $x \rightarrow n$
2. Construct truth tables for each statement:
 - a. $(p \rightarrow q) \iff (q' \rightarrow p')$
 - b. $(p \rightarrow q) \iff (p \vee q)$
3. From Exercise 7-9 we have $\sqrt{a} > 0$ and $(\sqrt{a})^2 = a$. Prove

$$x > 0 \text{ and } x^2 = a \implies x = \sqrt{a}.$$
4. Complete the formula using $|y - b| < 6 \iff y - b$.
5. True or false:
 - a. $S + O_2 \rightarrow SO_2 \uparrow$
 - b. $NaCl \rightleftharpoons Na^+ + Cl^-$
 - c. $HCl \rightleftharpoons H^+ + Cl^-$
 - d. $KI + I_2 \rightleftharpoons KI_3$
6. Show that $XY \subset RS$.
7. If two functions are equal for all values of n variables we can write

$$f(x_1, \dots, x_n) \equiv g(x_1, \dots, x_n)$$

This is an *identity*. Explain.
8. Argue that if $A \subset B$ and $B \subset C$, then $A \subset C$.
9. Argue that if $X \supset Y$ and $Y \supset Z$, then $X \supset Z$.
10. Let $M = \{1, 2, 3, \dots, 9\}$ and $N = \{2, 4, 6, 8\}$. Identify as true or false:
 - (a) $3 \in M$
 - (b) $3 + 4 \in N$
 - (c) $N \ni 8$
 - (d) $M \ni 9$
11. The vertices of a quadrilateral are $A(-1, 3)$, $B(-2, -1)$, $C(2, -2)$, and $D(3, 2)$. Prove:
 - (a) $AD \parallel BC$
 - (b) $AB \parallel DC$
 - (c) $AD \perp DC$
 - (d) $AC \perp BD$
12. Show that if a and b are elements of a set S , one and only one of the relations $a = b$, $a R b$, $b R a$ is true.
13. Is a relation $y \theta x$ a function if its graph has more than one y on any vertical line?
14. Show that $I \sim E_R^2$.
15. Coulomb's first experimental results are represented by

$$F \propto \frac{1}{r^2}.$$

Describe the method and apparatus he used.

What technique did he use to derive

$$F \propto \frac{q_1 q_2}{r^2} ?$$

16. Determine the solution sets if the universal set is the set of real numbers.

a. $\{m \mid 5(m + 2) = 10\}$

b. $\{v \mid -6(7v + 5) = 12\}$

17. Figure 32 shows all nuclei with $Z \gg 83$ are unstable. Explain.

18. Sketch the graphs of the following relations.

a. $x \geq -1$

b. $d \geq 2\frac{1}{2}$

c. $4x^2 + y^2 \leq 16$

d. $-3 \leq y - 2x \leq 3$

19. Prove $|w_1 + w_2| \leq |w_1| + |w_2|$.

20. Given the following sets

$$E = \{1, 3, 4, 6\}$$

$$F = \{1, 6\}$$

$$G = \{1, 3, 4, 6, 8, 9\}.$$

Label each statement as either true or false.

a. $E \subseteq F$

b. $G \subseteq F$

c. $G \supseteq F$

d. $E \supseteq G$

21. If X and Y are sets and we know that $7 \in X$ and $7 \in Y$, can we conclude $X \subseteq Y$? $Y \subseteq X$?

22. Show that two figures F and F' are projective if $F \overline{\wedge} F'$.

23. Make a statement about the truth value of p and q when $(p \vee q)$ is true and $(p \rightarrow q)$ is true.

24. Use Eq. (5) to find $(2.98)^3 \approx 26.46$.

25. Use the triangles in Figure 4.3 to prove $ACE \cong DBE$.

26. What is the effect of the assumption $x < = > b$ in the previous experiment?

LESSON 10

SIGNS OF SHAPE

§112. **Definition:** A sign of shape is a miniature picture of a geometric figure or another object.

\triangle (triangle)

\angle (angle)

\bigcirc (circle)

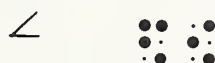
§113. **Shape Indicator**  **and Basic Signs of Shape:**

A basic sign of shape is represented by the shape indicator followed by one or more letters, by a numeral, or by a dot combination suggestive of the ink-print shape.

When the shape is a *regular polygon* (a closed figure with equal sides and equal angles), it is represented by the shape indicator followed by a numeral specifying the number of sides in the figure.

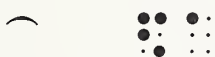
When the shape is an *irregular polygon* (a closed figure with at least two unequal sides and two unequal angles), it is represented by the shape indicator followed by a letter or a combination of letters suggestive of the ink-print shape.

Angle



Arc

Concave Upward



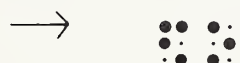
Concave Downward



Arrow

Right-Pointing

Contracted Form



Uncontracted Form



Left-Pointing



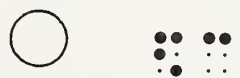
Down-Pointing



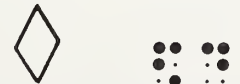
Up-Pointing



Circle



Diamond



Ellipse



Hexagon

Regular



Irregular



Intersecting Lines



Is Parallel To



Is Not Parallel To



Is Perpendicular To



Is Not Perpendicular To



Parallelogram



Pentagon

Regular



Irregular



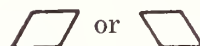
Quadrilateral



Rectangle



Rhombus



Square



Star



Trapezoid



Triangle

Regular (equilateral)



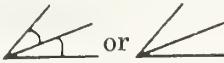







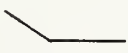


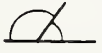

Inverted



§114. **Signs of Shape With Structural Modification:** A shape with structural modification is one in which two or more signs of shape are combined to form a composite sign, such as *adjacent angles*; or one in which the general print form of a basic shape is changed to show a more specific form, such as a *right angle*, a *right triangle*, etc.

A shape with structural modification is represented by the basic shape symbol followed by the structural shape-modification indicator $\begin{smallmatrix} \bullet \\ \vdots \\ \bullet \end{smallmatrix}$, by a letter or an uncontracted combination of letters suggestive of the change in the shape, and by the termination indicator $\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$ showing the end of the modification.



Angle

Adjacent Angles	 or 	$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Alternate Exterior Angles		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Alternate Interior Angles		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Complementary Angles		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Corresponding Angles		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Exterior Angles		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Interior Angles		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Obtuse Angle		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Right Angle		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Straight Angle		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Supplementary Angles		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Vertical Angles		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

Triangle

Acute		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Isosceles		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Obtuse		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Right		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$
Scalene		$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$

§115. **Signs of Shape With Interior Modification:** A shape with interior modification is a basic shape within which a letter, a numeral, a sign of operation, or other sign appears.

a. A shape with interior modification is represented by the basic shape symbol followed by the interior shape-modification indicator , by the symbol corresponding to the interior sign, and by the termination indicator  to show the end of the modification.

The numeric indicator must be used before a numeral or before a decimal point and a numeral following the interior shape-modification indicator.

If a right-pointing arrow in regular type with a full barb and single shaft of ordinary length is part of a shape symbol, its contracted form must not be used.

Angle

Angle With Interior Arc



Angle With Interior Clockwise Arrow



Angle With Interior Counterclockwise Arrow



Circle

Circle With Interior Arrow Pointing Right



Circle With Interior Arrow Pointing Left



Circle With Interior Arrow Pointing Up



Circle With Interior Arrow Pointing Down



Circle With Interior Capitalized Letter



Circle With Interior Numeral



Circle With Interior Cross



Circle With Interior Dot



Circle With Interior Minus Sign



Circle With Interior Plus Sign



Circle With Interior Vertical Bar



Rectangle With Interior Horizontal Bar



Square

Square With Interior Diagonal from Upper-Left to Lower-Right



Square With Interior Diagonal from Lower-Left to Upper-Right



Square With Interior Diagonals



Square With Interior Dot



Square With Interior Horizontal Bar



Square With Interior Vertical Bar



Square With Interior Numeral




b. When two or more vertically arranged modifiers occur within a basic sign of shape, the basic shape symbol and the interior shape-modification indicator must be followed by the symbols for the upper and then the next lower interior signs. The termination indicator must be used only after the last modifier symbol.

Circle With Interior Arrow Pointing Right
Over Interior Arrow Pointing Left



Circle With Interior Arrow Pointing Left
Over Interior Arrow Pointing Right



c. When two or more horizontally arranged modifiers occur within a basic sign of shape, the basic shape symbol and the interior shape-modification indicator must be followed by the symbols for the interior signs. The multipurpose indicator  must be placed between the interior modifiers to show that they are printed horizontally, not vertically. The termination indicator must be used only after the last modifier symbol.

Circle With Interior Arrow Pointing Up
Followed by Interior Arrow Pointing Down




Circle With Interior Arrow Pointing Down
Followed by Interior Arrow Pointing Up




§116. Unlisted Signs of Shape:

a. Basic signs of shape and shapes with structural modification not provided for in this code must be formed in accordance with the principles for the construction of such shapes. Contractions must not be used in any letter combination selected to represent a basic shape or its structural modification. A symbol which has been assigned a meaning must not be used. Any unlisted shape, except a regular polygon, must be explained by a transcriber's note giving the name or description of the symbol used. If possible, a drawing of the shape must be included.

(1)  (octagon)



(2)  (irregular 8-sided figure)



(3)  (moon)



(4)  (chicken)



b. Unlisted shapes with interior modification must be formed in accordance with the principles for the representation of such shapes. Transcriber's notes are required only for symbols that must be devised and are not listed in the code.

(1) 



(2) 




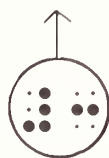
(3) 





§117. **Shapes Represented by Drawing:** Drawn-in shapes are often more readable than elaborate braille constructions. Since it is not possible to formulate specific rules for the selection of an appropriate form, the decision is left to the experience and judgment of the transcriber.

Shapes may also be represented by a combination of drawing and braille symbols. For example, if a modified shape cannot be represented clearly by braille symbols alone, the shape can be drawn, and the modification shown in braille.

(1) 



§118. **Filled-In and Shaded Shapes:** A filled-in or shaded closed shape (circle, diamond, square, etc.) must be represented as such by the *filled-in shape indicator*  or the *shaded shape indicator* . The appropriate indicator must be placed between the shape indicator and the shape symbol.

(1) 



(2) 



§119. Punctuation and Plurals With Signs of Shape:

a. Signs of shape must be punctuated mathematically.

(1) \angle , \triangle , \circ .



(2) $(\diamond, \nabla, \square)$

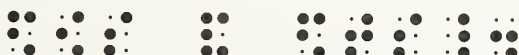


(3) \sphericalangle , \oplus .



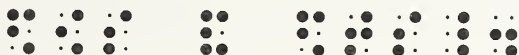
b. The uncapitalized letter “s” or the apostrophe-s combination occurring inside or after a sign of shape to show its plural or possessive must be placed after the shape symbol.

(1) \sphericalangle_s and \triangle_s .



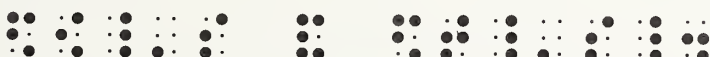
(Each s is inside the shape.)

(2) $\sphericalangle s$ and $\triangle s$.



(Each s follows the shape.)

(3) $\sphericalangle's$ and $\triangle's$.



(4) $(\sphericalangle's, \triangle's, \text{ and } \circ's.)$



§120. Spacing With Signs of Shape: A sign of shape must be spaced in accordance with its assigned meaning:

a. A sign of shape representing a sign of operation must be spaced accordingly.

The multipurpose indicator $\cdot\cdot$ must be used between a sign of operation represented by a regular polygon and a numeral following it.

(1) $x \oplus y$



(2) $2 \oplus 3 = 3 \oplus 2$



(3) $a \oplus (b \oplus c)$



(4) $(-a) \otimes (-b) = a \otimes b$



(5) $[a, b] \odot [a, b]$



(6) $a \oplus b \oplus \dots \oplus n$



(7) $r \otimes s \otimes \text{---} = rst$



(8) $x \square y = y \square x$



(9) $15 \square 15 = 30$



b. A sign of shape representing a sign of comparison must be spaced accordingly.

(1) $x \frown y$



(2) $f \rightarrow g$



(3) $BC \perp DE$



(4) $MN \parallel OP$



(5) $QR \not\parallel ST$

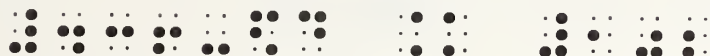


c. When a sign of shape is used as a sign of omission or placeholder to represent a numeral, letter, sign of operation, sign of comparison, abbreviation, or any other item, the sign of shape must be spaced as the material which it represents. In any case, a sign of shape must be unspaced from any braille indicator applying to it.

(1) $7 \times \triangle = 4 \times \triangle$ (numeral omitted)



(2) $436 - \diamond = 102$ (numeral omitted)



(3) $\triangle \times (\square - 46) = 46$ (numerals omitted)



(4) $N = (2 \times \diamond) + 1$ (numeral omitted)



(5) $\triangle + \square = \square + \triangle$ (numerals omitted)



(6) $(\square + \triangle) + \diamond = \square + (\triangle + \diamond)$ (numerals omitted)



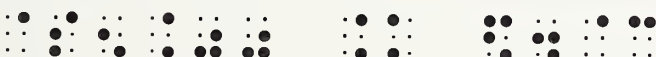
(7) $\{\triangle, \square, \bigcirc\} \cup \{\square\}$ (numerals omitted)



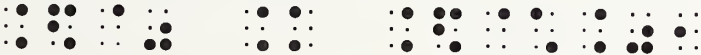
(8) $40 \text{ dimes} = \$\triangle$ (numeral omitted)



(9) $\$5.00 = \square\phi$ (numeral omitted)



(10) $\nabla \% = \nabla \times .01$ (numeral omitted)



(11) $\frac{15}{20} = \frac{\square}{\bigcirc}$ (numerals omitted)



(12) $8\frac{2}{6} = 8\frac{\triangle}{3}$ (numeral omitted)



(13) $\square_8 + \square_8 = 22_8$ (numeral omitted)



(14) $x^2 \times x^4 = x^{\square}$ (numeral omitted)



(15) $24_{\diamond} + 11_{\diamond} = 40_5$ (numeral omitted)



(16) $\square \text{ l.} = 1000 \text{ cc.}$ (numeral omitted)



(17) $11 \text{ yds. } 16 \text{ ft.} = \square \text{ yds. } \diamond \text{ ft.}$ (numerals omitted)



(18) $3 \diamond 7 = 10$ (sign of operation omitted)



(19) $50 \blacksquare 50 = 100$ (sign of operation omitted)



(20) $\frac{1}{4} \circ \frac{1}{4} = \frac{1}{2}$ (sign of operation omitted)



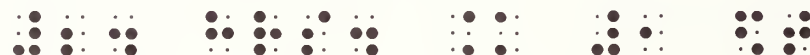
(21) $50 + 50 \circ 100$ (sign of comparison omitted)



(22) $99 \text{ } \text{diagonal lines} \text{ } 100$ represents $99 < 100$ (sign of comparison omitted)



(23) $24 \text{ hrs.} = 1 \triangle$ (abbreviation omitted)



§121. Identified Signs of Shape:

- a. A space must be left between a sign of shape and its identifying numeral, letter, or sequence of letters following it.

(1) $\angle \alpha$



(2) $\angle 1$



(3) $\triangle ABC$ is a right triangle.



(4) $\square EFGH$ is a parallelogram.



(5) Angle (\angle) BAC is a right angle.



(6) \triangle 's PQR and STU are similar.

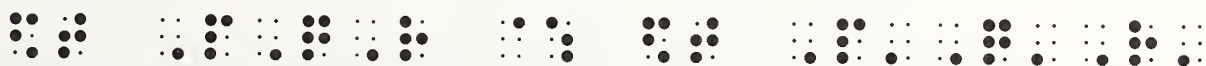


b. The spacing before and after a sign of shape and its identification is subject to the spacing rules for the symbols preceding or following it.

(1) $\angle 1 < \angle 2$



(2) $\triangle PQR \sim \triangle P'Q'R'$



(3) $\triangle ADM \cong \triangle BEM$



(4) $\triangle II \sim \triangle II'$



(5) $\angle 2 + \angle 3 = \angle 4$



(6) $\angle x + \angle y$



(7) $\angle ABD + \angle DBE = ?$



(8) $\angle 45^\circ + \angle 45^\circ = ?$



(9) $(\angle 1 + \angle 2) + (\angle 3)$



$$(10) \frac{\triangle ABC}{\triangle DEF}$$



$$(11) \triangle EFG / \triangle RST$$



$$(12) \angle ECB = \frac{1}{2} \angle ABC$$



$$(13) m \angle 2 + m \angle 3 = m \angle 4$$



$$(14) m \angle ABC + m \angle CDE$$



$$(15) m^\circ \angle \alpha = -42^\circ$$



c. The effect of the level indicator before an identified sign of shape in a superscript or subscript extends through the space following the sign of shape. In such cases, the space preserves the superscript or subscript level where the sign of shape appears. Thus, the identifying symbol following the space assumes the same level as its related sign of shape.

$$(1) A_{\triangle ABC}$$



$$(2) A_{\triangle DEF} = \frac{1}{2} bh$$



$$(3) A_{\square ACIJ} + A_{\square BGNC}$$



$$(4) (I_{\angle PMQ}) \cup (\angle PMQ)$$



d. When an identified sign of shape carries a superscript or subscript, the space required after the sign of shape must follow the superscript or subscript. If the sign of shape is on the baseline of writing, the space following the superscript or subscript terminates the effect of the level indicator and reinstates the baseline level where the sign of shape appears.

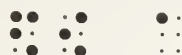
$$(1) \triangle^2 DEF$$



§122. Signs of Shape and the English Letter Indicator:

a. The English letter indicator must not be used with an English letter, a short-form combination, or a Roman numeral in regular type after a space following a sign of shape. However, if the sign of shape has a plural or possessive ending, the general rules for the English letter indicator apply.

(1) $\angle a$



(2) $\perp B$



(3) $\odot 0$ denotes "circle O."



(4) $\angle a = \angle b$



(5) $\angle b = 3 \angle y$



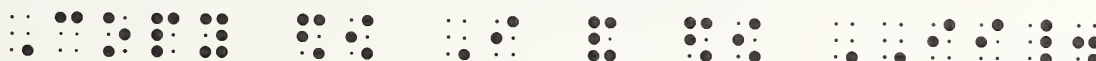
(6) $\angle A = 2(\angle B)$



(7) $\triangle dcl$ and $\triangle DCL$ are similar.



(8) Copy $\angle I$ and $\angle II$.



(9) $\triangle i$ and $\triangle ii$ are not equal.



(10) \angle 's A and B are acute.

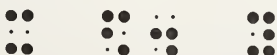


(11) \triangle 's I and II are similar.



b. The English letter indicator must not be used with an English letter, a short-form combination, or a Roman numeral in regular type followed by a space which in turn precedes a sign of shape used as a sign of omission. If the sign of shape is not a sign of omission, the general rules for the English letter indicator apply.

(1) $x \square y$



(the symbol for the "square" is used as a sign of omission)

- (2) Find the sum of the n \angle 's.



(the symbol for the “angle” is not an omission sign)

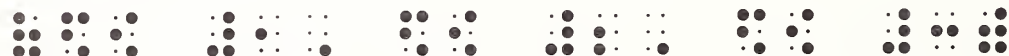
§123. Signs of Shape With Enclosed Lists: In an enclosed list, an identified sign of shape and the following numeral, letter, or sequence of letters must be considered as a single item, even though a space occurs between them.

The numeric indicator must not be used before a numeral or before a decimal point and a numeral in regular type at the beginning of an item in an enclosed list; however, it must be used when these symbols follow an identified sign of shape.

- (1) (\angle 1, 2 \angle 1, 3 \angle 1)



- (2) (\angle 1, \angle 2, \angle 3)



- (3) (1 \angle a, 2 \angle b)



HOMEWORK

Prepare the following homework for submission to your teacher. Proofread carefully.

EXERCISE 10

- \angle , \circ , \diamond , and \square are all figures we will study.
- How do we find the area of a \circ ?
- Copy each shape and explain its meaning.

a.

b.

c.

d.

e.

f.

g.

4. Add $\angle 30^\circ$ and $\angle 20^\circ$.

5. Substitute \square s for the \bigcirc s:
 $(\bigcirc, \square, \square, \triangle, \bigcirc).$
6. (\diamond 's, ∇ 's, and \triangle 's.)
7. \bigcirc_s and \square_s are geometric shapes.
8. If ABCD is a parallelogram and CDFE is a parallelogram, prove:
 - (a) ABEF is a \square .
 - (b) $FA = EB$ and $FA \parallel EB$.
9. If $\square + \triangle = N$, does this mean that $\square = N - \triangle$? Does it also mean that $\triangle = N - \square$?
10. Write $+$ or $-$ in each shape.
 - a. $5\triangle = 7$
 - b. $9\triangle = 8$
 - c. $10\triangle = 7$
 - d. $3\square = 5$
 - e. $(a-b) \square a = -b$
11. Write $>$, $<$, or $=$ in each shape.
 - a. $3+5 \bigcirc 7$
 - b. $8-1 \bigcirc 10$
 - c. $4+5 \bigcirc 6+2$
 - d. $8-6 \square 5+1$
 - e. $4-1 \square 2+2$
12. $(3 \otimes 2) + 1 = 7$
13. If $250¢ = \$2.50$, then $736¢ = \$\bigcirc$. Similarly, we see that if $\$6.00 = 600¢$, then $\$9.32 = \square¢$.
14. 148 people went to a snack bar. 111 were grown-ups. $\triangle\%$ were children.
15. a. $\frac{192}{80} = \frac{\triangle}{40} = \frac{\bigcirc}{\square}$
 b. $23\frac{15}{25} = 23\frac{\square}{5}$
16. Using the "common factor" method, we see that $x^{n+3} - 5x^n = x^\bigcirc (x^\triangle - 5)$.
17. 1 qt. = \square pt. and 4 qt. = \bigcirc pt. How are quarts related to pints?
18. Solve the problems.
 - a. 2 ft 3 in = 27 \square
 - b. 2 yd = 6 \bigcirc
 - c. $\frac{1}{4}$ ft. = \square in.
19. $\triangle ABD \cong \triangle ACE$
20. In $\triangle ABCD$, $BC = AD$. If diagonals AC and BD intersect at E, prove that $\triangle DEA \sim \triangle CEB$.

21. If ABCD is an isosceles trapezoid, $DC \parallel AB$, $DE \perp AB$, and $CF \perp AB$, prove that $\triangle DEA \cong \triangle CFB$.
22. Is $\angle B$ acute or obtuse?
23. In $\triangle ABC$, $\angle A = 3\angle C$, and $\angle B = 36^\circ$. Find the number of degrees in $\angle C$.
24. $\angle a + \angle b + \angle c = 180^\circ$
25. If $\triangle ABC = \frac{1}{2} \square ABEC$ and the area of $\square ABEC = bh$, then the area of $\triangle ABC = \frac{1}{2} bh$.
26. Find the sums.
 - a. $\angle 30^\circ + \angle 25^\circ = ?$
 - b. In $\triangle ABC$, $m\angle A + m\angle B + m\angle C = ?$
27. What does $\square^2 ABCD$ mean?
28. Prove $A_{\square ABCD} = A_{\triangle ADE}$.
29. Draw a figure illustrating the following statement:
 $A_{\triangle 1} + A_{\triangle 2} = A_{\triangle 3}$.
30. Show that $\angle acr$ and $\angle afn$ are equal.
31. $\angle C = \angle D$ and \angle 's C and D are both obtuse.
32. $\bigcirc i$ and $\bigcirc ii$ have diameters of the same size.
 \bigcirc 's i and ii therefore also have equal radii.
33. If $y > 0$, then $x \square x + y$. (Use $>$, $<$, or $=$.)
34. $(\angle 5, \angle 6, \angle 7)$.
35. $(5\angle 3, 2\angle 2, 6\angle 1)$.

LESSON 11

MODIFIERS AND MODIFIED EXPRESSIONS

§124. **Definition:** A modifier is a symbol or a combination of symbols occurring directly over or under its related symbol or expression.

$$\overline{34} \quad \widetilde{x} \quad \underline{x + y}$$

The most commonly used modifiers are listed below. Other symbols of the code may also be used as modifiers.

Modification Indicators

Multipurpose Indicator	⠆
Directly-Over Indicator	⠈
Directly-Under Indicator	⠊
Termination Indicator	⠋

Modifiers







































Arc

Concave Upward	⌒	⠠
Concave Downward	⌓	⠡

Arrow

Barbed at Right

Contracted Form	→	⠶
Uncontracted Form	➔	⠶⠶⠶⠶
Barbed at Left	←	⠷
Barbed at Both Ends	↔	⠸
Barbed at Right and Dotted at Left	→•	⠶⠠
Barbed at Left and Dotted at Right	←•	⠷⠡
Dotted at Right (no barb)	→•	⠶⠡

Dotted at Left (no barb)		
Dotted at Both Ends		
Hollow Dot at Right (no barb)		
Hollow Dot at Right and Barb at Left		
Hollow Dot at Left (no barb)		
Hollow Dot at Left and Barb at Right		
Hollow Dot at Both Ends		
Bar		
Horizontal (macron)		
Vertical		
Caret		
Circumflex		
Inverted		
Left-Pointing		
Right-Pointing		
Dot		
Dot		
Hollow Dot		
Question Mark		
Tilde		
Single		
Extended		
Triangle (equilateral)		

§125. Construction of Simple Modified Expressions:

a. The Five-Step Rule: The following five-step rule must be used for the transcription of a modified expression. (See exception in **c** below.)

- i. The multipurpose indicator $\begin{smallmatrix} \cdot \cdot \\ \cdot \cdot \end{smallmatrix}$ must be placed immediately before the expression to be modified.
- ii. The expression to be modified must be written second.
- iii. The directly-over indicator $\begin{smallmatrix} \cdot \cdot \\ \cdot \cdot \end{smallmatrix}$ or the directly-under indicator $\begin{smallmatrix} \cdot \cdot \\ \cdot \cdot \end{smallmatrix}$ must be written third to show the position of
- iv. The modifying symbol must be written fourth.
- v. The termination indicator $\begin{smallmatrix} \cdot \cdot \\ \cdot \cdot \end{smallmatrix}$ must be written last to show the end of the modification.

b. The spacing before and after a modified expression is subject to the spacing rules for the symbols preceding or following it. Two or more symbols in a modifier must be spaced in accordance with their appropriate spacing rule.

(1) $\overline{34}$

(2) $1.142857\overline{142857}$

(3) $1.\overline{37}$

$$(4) \quad \frac{3}{7} = \overline{.428571}$$

(5) \overline{PQ}

$$(6) \quad \overline{OP} + \overline{QR} = \overline{OR}$$

$$(7) \quad \overline{AB^2} + \overline{BC^2}$$

(8) $\overline{x'}$

(9) $\overline{R''S''}$

$$(10) \quad m\overline{BC} = a$$



$$(11) \quad \frac{\overline{PR}}{\overline{OR}}$$



$$(12) \quad \overline{x+y}$$



$$(13) \quad \tilde{u} = 0.8$$



$$(14) \quad f \longrightarrow \tilde{f}$$



$$(15) \quad \tilde{A} \cup \tilde{B}$$



$$(16) \quad \widetilde{x + y}$$



$$(17) \quad \widetilde{(A \cap B \cap C)}$$



$$(18) \quad \widetilde{r+s+t}$$



$$(19) \quad \angle ABD = \frac{1}{2} \widehat{DC}$$



$$(20) \quad \underline{x}$$



$$(21) \quad \underline{3}$$



$$(22) \quad \underline{x+y}$$



$$(23) \quad A(\underline{\text{sn}})$$



$$(4) \quad 2 \cdot 3 = \overline{2} \cdot 3 = \overline{2 \cdot 3}$$



$$(5) \quad \overline{C} = 100 \times 1000$$



$$(6) \quad \overline{s}$$



$$(7) \quad \overline{\alpha}$$



$$(8) \quad F = 2 \pi \overline{r} l$$



$$(9) \quad P(\overline{x}) = \overline{P(x)}$$



$$(10) \quad g(\overline{xy}) = g(\overline{xy})$$



$$(11) \quad P(\overline{a+bi}) = \overline{0} = 0$$



$$(12) \quad \overline{x'}$$



$$(13) \quad \overline{x^2}$$



$$(14) \quad a_0 \overline{x^n} + a_1 \overline{x^{n-1}}$$



$$(15) \quad \overline{A} = [\overline{a_i}]$$



$$(16) \quad (\overline{x}, \overline{y})$$



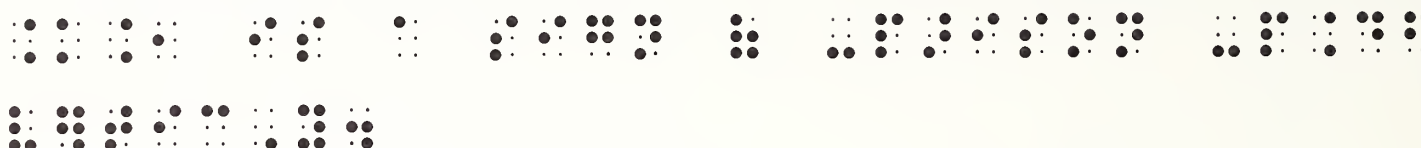
(5) $\overline{\overline{7}} \cdot \overline{\overline{2}} = \overline{\overline{7 \cdot 2}}$



(6) $\overline{\overline{x}}$



(7) $\overline{>}$ is a sign of comparison compounded vertically.



b. When two or more modifiers do not apply to exactly the same symbols but cover different portions of the same expression, the longer modification must enclose the entire modified expression, and each inner expression must be modified individually. The five-step rule and the contracted form of transcription may be used together.

(1) $\overline{(\overline{aA} + \overline{bB})}$



(2) $\overline{7} + 2 = \overline{7 + 2}$



(3) $\overline{A \cap B \cap \overline{C}}$



(4) $\overline{\overline{x \times x}}$



§127. **Simultaneous Modifiers:** When an expression is simultaneously modified both above and below, the modifier below must be shown first and the modifier above, second. However, the termination indicator must be used only at the end of the entire modification.

(1) $\overline{\underline{x}}$



(2) $\overline{\underline{x + y}}$



(3) $\overline{\underline{N}}$



$$(4) \sum_{k=1}^n (2k) = n(n+1)$$



$$(5) \prod_{k=2}^6 a_k$$



$$(6) \sum_{i,j=1}^n a_{ij} x_i x_j$$



$$(7) \sum_{i=1}^k \sum_{j=1}^k$$



$$(8) \sum_{x_1=0}^1 \sum_{x_2=0}^1$$



$$(9) \bigcup_{r=1}^n A_r = A_1 \cup A_2 \dots$$



$$(10) \overline{\prod_{x \in a} c'x}$$



$$(11) \prod_{\substack{j=1 \\ j \neq k}}^n$$



$$(12) \frac{b=2}{x+y} \quad \frac{a=3}{}$$



§128. Modified Expressions and Superscripts and Subscripts:

a. A modifier and its related expression must always be placed on the same level of writing. When a modifier affects both an expression on the baseline of writing and its superscript or subscript, the baseline indicator must be used after the superscript or subscript and before the directly-over or the directly-under indicator to indicate the level where the expression as a whole appears. However, it is not necessary to indicate a return to the baseline after a numeric subscript not requiring a subscript indicator.

Note: The contracted form of transcription must not be used when a horizontal bar extends above a superscript or subscript.

$$(1) \overline{x^2}$$



$$(2) \quad \overline{\overline{A^n}}$$



$$(3) \quad (\overline{3^{-1}}) \in P$$



$$(4) \quad \overline{OD^2} + \overline{OP^2}$$



$$(5) \quad \overline{x_n}$$



$$(6) \quad \overline{P_n P_{n+1}}$$



$$(7) \quad \sum_{i=1}^6 \overline{P_{i-1} P_i}$$



$$(8) \quad \overline{x_1}$$



$$(9) \quad \overline{x_1} + \overline{y_1}$$



$$(10) \quad \overline{P_1 P_2}$$



$$(11) \quad \overline{P_1 Q^2} + \overline{Q P_2^2}$$



$$(12) \quad R_1 - R_2 = \overline{(R_1 - R_2)}$$



$$(13) \quad \underline{Z^\circ}$$



$$(14) \quad \underline{N_k}$$



$$(2) \tilde{S}_{\tilde{x}+\tilde{y}}$$



$$(3) S_{\tilde{x}}$$



$$(4) \tilde{S}_{\tilde{x}+\tilde{y}}$$



$$(5) \tilde{x}A_1$$



$$(6) \overline{P_{(x^2)}}$$



$$(7) A^{\tilde{x}}$$



$$(8) e^{a\tilde{x}}$$



$$(9) A_{\tilde{x}}$$



$$(10) A_{\tilde{x}+\tilde{y}}$$



$$(11) 3_{\tilde{x}}-2_{\tilde{x}}$$



$$(12) \tilde{n}A_1$$



$$(13) \overline{a_{\tilde{n}}+b_{\tilde{p}}}$$



$$(14) \overline{x_{\tilde{n}}+y_{\tilde{n}}}$$



§129. Arrows as Modifiers:

a. An arrow must be treated as a modifier when it occurs directly over or directly under a symbol other than a sign of comparison. An arrow which occurs above or below a sign of comparison must be treated as part of a sign of comparison compounded vertically (see §108).

When a right-pointing arrow in regular type with a full barb and single shaft of ordinary length is used as a modifier above or below a mathematical expression, the arrow must be transcribed in its contracted form. However, if such a right-pointing arrow is itself modified, or is part of a compound modifier, its contracted form must not be used. All other arrows require all their appropriate symbols.

(1) \overrightarrow{AB}



(2) \overrightarrow{T}



(3) $\overrightarrow{AB} - \overrightarrow{CD}$



(4) $|\overrightarrow{O_1P}| + |\overrightarrow{P_1P_2}|$



(5) $\lambda \overrightarrow{BA} + \lambda \overrightarrow{BC}$



(6) $x \xrightarrow{\text{fog}} y$



(7) $x \xrightarrow{g} y \xrightarrow{f} z$



(8) \overrightarrow{F}



(9) \overleftarrow{AB}



(10) $\overleftrightarrow{XZ} \parallel \overleftrightarrow{RS}$



(11) $\overleftrightarrow{AB} + \overleftrightarrow{CD}$



(12) \overleftarrow{AB}



$$(13) \quad \overrightarrow{OB} \cup \overrightarrow{OC}$$

(14) \overline{AB}

A 3x12 grid of dots representing a 3x12 Latin square. Each column contains a unique 3x3 subgrid of dots, and each 3x3 subgrid appears exactly once in the columns.

(15) \overline{AB}

(16) \overline{AB}

A 3x12 grid of dots representing a 3x12 Latin square. Each column contains a unique 3x3 subgrid of dots, and each 3x3 subgrid appears exactly once in the columns.

(17) \overline{AB}

(18) T

b. When identical arrows are used above vectors in boldface type throughout the text, they must be omitted from the braille transcription unless the author specifically refers to them as a notational device. If the arrows are to be omitted in braille, a transcriber's note must be included explaining that the arrows are present in print.

$$(1) \quad \mathbf{b} + \mathbf{a} = \vec{\mathbf{OP}}$$

(OP is in boldface type, arrow omitted, T.N. required)

$$(2) \quad \mathbf{b} + \mathbf{a} = \vec{OP}$$

[illegible]

(OP is in ordinary type; arrow shown; T.N. not required)

(3) $\mathbf{v}[\overrightarrow{\mathbf{EF}}]$ and $\mathbf{r}[\overrightarrow{\mathbf{st}}]$

(Both arrows must be shown since they are not of uniform construction. T.N. not required)

§130. Dots as Modifiers:

a. In print the recurrence of one or more digits in a decimal numeral may be indicated by one dot over each recurring digit. However, in braille only one dot is used as a modifier. The dot is placed after the last modified digit in the recurring sequence.

(1) $\dot{.4}$



(2) $\frac{1}{6} = 0.166\dot{6} \dots$



(3) $1.\ddot{3}75$



(4) $\frac{1}{11} = 0.9090\ddot{9}$



b. When one or more dots occur over or under a single mathematical expression, the symbol for the dot must be used as many times as necessary to conform with the print text.

(1) $\ddot{x} = \frac{d^2x}{dt^2}$



(2) \ddot{x}



(3) $\ddot{xy} - \ddot{yx}$



(4) $\sqrt{\ddot{x}^2 + \ddot{y}^2}$



(5) \ddot{x}



§131. **Horizontal Grouping Signs as Modifiers:** A horizontal grouping sign over or under a mathematical expression must be treated as a modifier. It is recommended that such grouping signs be drawn.

However, horizontal grouping signs may also be represented by braille symbols, in which case the entire modified expression must be transcribed in accordance with the five-step rule. The opening sign of grouping must be used when the modifier is above, and the closing sign of grouping when the modifier is below.

(1)

$$\overbrace{x + y}$$

(2)

$$\underbrace{x + y}$$

(3)

$$\overbrace{x + y}$$

(4)

$$\underbrace{x + y}$$

(5)

$$\overbrace{.000 \dots 016617}^{23 \text{ zeros}}$$

§132. **Modified Signs of Comparison:** A modified sign of comparison consists of a simple sign of comparison, such as the equals sign or the tilde, modified by a caret, dot, triangle, question mark, vertical bar, or any symbol except another sign of comparison. Such a combination must be transcribed in accordance with the five-step rule for modified expressions. However, when a simple sign of comparison occurs above or below another simple sign of comparison, the combination must be transcribed as a sign of comparison compounded vertically, and the five-step rule must not be used (see §108).

A modified sign of comparison may also consist of a horizontal bar modified by a dot under it or by a caret directly over or under it. However, if the horizontal bar is modified by a dot over it, the combination is a sign of operation (see §94j).

The following list contains the modified signs of comparison most commonly used. Unlisted modified comparison signs must be transcribed in accordance with the same principles.

Equals Sign

Caret Over	$\overset{\wedge}{=}$	
Caret Under (is projective to)	$\underset{\wedge}{=}$	
Inverted Caret Over	$\overset{\vee}{=}$	
Left-Pointing Caret Over	$\overset{<}{=}$	
Right-Pointing Caret Over	$\overset{>}{=}$	
Dot Over (is approximately equal to)	$\overset{\cdot}{=}$	

Dot Over and Dot Under



Two Dots Over and Two Dots Under



Hollow Dot Over (is equal in degrees to)



Equilateral Triangle Over



Question Mark Over



Question Mark Under



Vertical Bar Over



Simple Tilde

Dot Under



Dot Over



Horizontal Bar

Caret Over



Caret Under (is perspective to)



Dot Under



(1) $\sqrt{3} \doteq 1.732$



(2) $A \doteq 3.14r^2$



(3) $\angle b \overset{\circ}{=} \frac{1}{2} \widehat{EB}$



(4) $5(3) + 2(-2) \overset{?}{=} 11$



(5) $A \overset{D}{=} B$



MULTIPURPOSE INDICATOR

Multipurpose Indicator



§133. **Review of the Multipurpose Indicator:** The multipurpose indicator has already been introduced in the following situations (in these situations it must not be regarded as the baseline indicator):

- a. The multipurpose indicator must be used between two symbols for the tilde to indicate that they are written horizontally (see §94k).
- b. The multipurpose indicator must be used between two unspaced signs of comparison to indicate that they are printed horizontally and not vertically (see §109).
- c. The multipurpose indicator must be used between two horizontally arranged modifiers within a sign of shape (see §115c).
- d. The multipurpose indicator must be used between a regular polygon representing a sign of operation and a numeral immediately following it (see §120a).
- e. The multipurpose indicator must be used before a modified expression (see §125a.)

§134. Additional Uses of the Multipurpose Indicator:

a. When both a letter and a numeral following the letter appear on the baseline of writing, the multipurpose indicator must be used before the numeral to show that it is not a subscript to the letter. However, a letter used to represent a numeral in a nondecimal numeration system must be regarded as a numeral, and the multipurpose indicator must not be used.

(1) x3



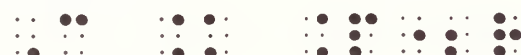
(2) R10



(3) $4\theta = 120^\circ + n360^\circ$



(4) $C = \pi 2r$



(5) $\omega 2 = \omega \neq 2\omega$



(6) Serial number GE5678H9



(7) 2TE14₁₂ is a base-12 numeral.



(8) $140TE4T5_{12} + E5_{12}$



b. When a letter on the baseline of writing is immediately followed by a decimal point and a numeral, the multipurpose indicator must be placed between the letter and the decimal point to show that the decimal point and numeral are not subscripts to the letter.

(1) $x.4$



c. The multipurpose indicator must be used after a numeric subscript if the subscript is followed by a numeral on the baseline of writing.

(1) x_710



(2) $\frac{A_0}{2} = A_0 2^{-0.05T}$



(3) $x_2 = n_1 5^{-1} - 1 n_2 5^{-1}$



d. The multipurpose indicator must be used after a decimal point if the symbol following the decimal point is *not* a numeral. However, the multipurpose indicator must not be used when the decimal point is followed by a comma or the punctuation indicator.

(1) $0.a_1 a_2 a_3 \dots a_n$



(2) $0.\alpha_1 \alpha_2 \alpha_3 \dots \alpha_n$



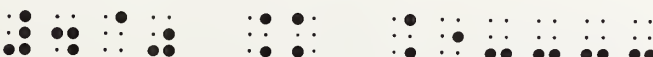
(3) $.%$



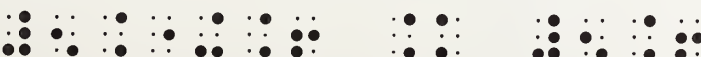
(4) $\frac{1}{2} = . \underline{\quad ?}$



(5) $4\% = .\underline{\hspace{1cm}}$



(6) $5. + .6 = 5.6$



(7) (5.)



(8) Can you explain 1., 10., and 100.?



e. The multipurpose indicator must be used between two unspaced vertical bars when the first bar is a closing sign of grouping, and the second bar is an opening sign of grouping.

The multipurpose indicator must also be used between two vertical bars which are grouping symbols when one bar is shorter and/or thicker than the other.

(1) | x | | y |



(2) || x || || y ||



(3) | | x | |



(Outside bars are longer than inside bars.)

(4) || | x | ||



(Inside bars are longer than both sets of outer bars.)

(5) | x | |
x = 0



(Third bar extends to the subscript level.)

HOMEWORK

Prepare the following homework for submission to your teacher. Note: the transcriber's note required on page 206 should be placed at the appropriate point in the transcription. Proofread carefully.

EXERCISE 11

Find the fractional equivalent of each repeating decimal.

1. $2.43\overline{43}$

2. $0.23678\overline{78}$

3. $1.142857\overline{142857}$

4. $.249\overline{9}$

5. $7.13\overline{13} \dots$

6. $6.253\overline{253} \dots$

7. In quadrilateral ABCD, $\overline{DC} \parallel \overline{AB}$, and \overline{AB} equals \overline{DC} . The measures of certain angles are represented by a, b, and c. Write the proof that $b = c$.
8. In \square DEFG, $m\overline{DE} = 75$ and $m\overline{EF} = 20$. If $m\overline{HJ} = m\overline{EF}$ find $m\overline{HJ} + m\overline{DE}$.
9. In the figure given, if $\widehat{AB} = \widehat{CD}$ in circle O, then $\angle AOB = \angle BOC$.
10. Does $\frac{\overline{ST}}{\overline{UV}}$ equal $\frac{\overline{UV}}{\overline{ST}}$?
11. In right triangle ABC, if \overline{AB} is the hypotenuse, is it true that $\overline{BC}^2 + \overline{AC}^2 = \overline{AB}^2$?
12. Using workbook pages 159-161, explain the following:
 - a. $\widetilde{r} = 72.9$ (page 159)
 - b. $\widetilde{T} \cup \widetilde{V}$ (page 160)
 - c. $\widetilde{a+b+c}$ (page 161)
13. Let the functions f and g be defined as $f: \underline{\mathbb{R}} \rightarrow \underline{\mathbb{R}}$ with $f(x) = 2x+1$, $g: \underline{\mathbb{R}}^+ \rightarrow \underline{\mathbb{R}}^+$ with $g(x) = x^{\frac{1}{2}}$.
14. Explain the meaning of $\underline{\mathbb{C}}^*/\underline{\mathbb{R}}^*$.
15. Give the place value of the underlined material.
 - a. 3.579
 - b. 62.175
 - c. 0.000001
16. Find the fractional equivalent of each repeating decimal.
 - a. $0.11\overline{1}$
 - b. $2.44\overline{4}$
17. $\overline{x} \times \overline{y} = \overline{xy}$
18. $\overline{\alpha}$ and $\overline{\beta}$ identify the circles.
19. The volume V of a right circular cone is given by the formula

$$V = \frac{1}{3} \overline{A}h,$$
 where \overline{A} is the area of the base.
20. The volume of a sphere varies directly as the cube of its radius, since

$$V = \frac{1}{3} \pi \overline{r}^3.$$
21. $z = m \frac{\overline{x^2}}{\overline{y^3}}$
22. $\frac{\overline{p_x}}{\overline{p_y}}$ should be used for checking your work.
23. How many $\overline{6}$'s are needed to fill the blanks in the above example?
24. Is it true that the arbitrary set $\overline{[A \cap X \cup (B \cap \overline{X})]}$ is equal to $(\overline{A} \cap X) \cup (\overline{B} \cap \overline{X})$?
25. All values of y are located on the regression line $\hat{y} = \alpha + \beta x$.

26. The probability of the event A, written $P(A)$, is defined as

$$P(A) = \sum_{\Lambda} f(x)$$

where $\sum_{\Lambda} f(x)$ means sum $f(x)$ over those values x_i that are in A.

27. The variance is the sum of $\sum_i \alpha_i^2 \sigma_i^2$ with the product of $\sum_{i \neq j} \sum_j$ and $\alpha_i \alpha_j \sigma_{ij}$.

28. Can you obtain the maximum likelihood estimators $\hat{\bar{r}}$ of the \bar{r}_{ij} by replacing the θ_i by $\hat{\theta}_i$?

29. Explain $\bar{y} \times \bar{z}$ in words.

30. $\sum_{\substack{i, j, k \\ i < j < k}}$

31. A sum of terms such as $n_3 + n_4 + n_5 + n_6 + n_7$ is often designated by the symbol $\sum_{i=3}^7 n_i$.

32. The probability of A_i is $\sum_{j=1}^s \frac{n_{ij}}{n}$.

33. Mark point $(\hat{\mu}, \hat{\sigma}^2)$ on the graph in Figure 11.2.

34. To get the reduced normal equations, cross the row and column corresponding to $\hat{\beta}_3$ from $X'X$ and from $X'Y$.

35. If $\bar{a}_1 = 72$, find \bar{a}_7 .

36. $W = \frac{2}{3} \pi r^3 w (h + \frac{3}{8} r)$.

37. If $a^{\tilde{n}} \cdot a^{\tilde{m}}$ equals $a^{\tilde{n}+\tilde{m}}$, what is the product of $x^{\tilde{3}}$ and $x^{\tilde{2}}$? What is the product of $x^{\tilde{7}}$ and $x^{\tilde{9}}$?

38. Let A, B, C, D be the vertices, in order, of a quadrilateral. Let A', B', C', D' be the mid-points of the sides \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} , in order. Prove that A'B'C'D' is a parallelogram. (Hint: First show that $\overrightarrow{A'B'}$ equals $\overrightarrow{D'C'}$ equals $\frac{1}{2} \overrightarrow{AC}$.)

39. $\overrightarrow{P_1 P_2}$ if P_1 is the point (1, 3) and P_2 is the point (2, -1).

40. $\overrightarrow{OP_3}$ if O is the origin and P_3 is the mid-point of the vector $\overrightarrow{P_1 P_2}$ joining P_1 (2, -1) and P_2 (-4, 3).

41. R equals \overrightarrow{OP} equals \overrightarrow{OM} plus \overrightarrow{MC} plus \overrightarrow{CP} .

42. $\overrightarrow{OP} = ix + jy$.

43. \overrightarrow{OP} equals \overrightarrow{OT} plus \overrightarrow{TP} .

44. What is the fractional equivalent of $2.431\dot{3}\dot{1}$?

45. Give the sum of \ddot{a} and \ddot{b} .

46. $\underset{\cdot}{a} + \underset{\cdot}{a} = ?$

47. $x^n = \underbrace{x \cdot x \cdot x \cdot \dots \cdot x}_n$

48. $\frac{48}{48} = \frac{?}{4}, 1 = 1.$

49. Find the surface area of a sphere if $S = 4\pi r^2$, $\pi = \frac{22}{7}$, and $r = 21$ cm.
50. If 2793t4e6 is a numeral in base-twelve, what is its equivalent in base-ten?
51. If $z.6$ equals $7\frac{3}{5}$, what do you know about z ?
52. $D_8 6_3 = Y^2$
53. The orbital speed of the planet Mercury is 29.73 miles per second and the orbital speed of Jupiter is 8.11 miles per second. The difference is $29.73 - 8.11 = \square.\triangle$.
54. Does $|-2| + |3| = +6$? Does $|-92| + |-5| = -460$?
55. $\binom{n}{k} = C_n^k$

LESSON 12

MISCELLANEOUS SYMBOLS

§135. **Unspaced Miscellaneous Symbols:** No space should be left between the symbols listed below and any other symbol or quantity to which they apply. However, a space must be left between these symbols and a word, an abbreviation, a sign of comparison, or other symbol which specifically requires a space before or after it.

a. **Caret (circumflex):** \wedge $\begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \end{array}$

The caret may be used as a modifier above or below a mathematical expression or as a place indicator with the decimal point (to show the position to which the decimal point has been moved). Used as a place indicator, it must be treated as a numeric symbol.

(1) .37_Λ68

•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•

(2) 1.2_A

(3) $\wedge x$

(4) The caret (^) shows the place to which the decimal is moved.

b. Crossed d: \overline{d} $\begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \end{array}$

Crossed h: ħ ⚏ ⚛

Crossed Lambda: λ $\begin{array}{ccc} \bullet & \bullet & \bullet \\ \vdots & \vdots & \bullet \\ \vdots & \vdots & \bullet \end{array}$

Crossed R: R_{c} $\begin{array}{ccc} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array}$

(1) $2d_v$

• • • • •

$$(2) \quad 2\pi\hbar\nu = \phi e$$




(3) $(v > \phi \hbar)$

(4) $\hbar = 6.625 \times 10^{-27}$ erg sec.

(5) $3\chi \dots n\chi$



(6) $\frac{h}{R}$



(7) $n = 3R$




(8) $R(P_1P_2, P_3P_4)$



(9) $R: 24 \text{ grams}$



c. Del (nabla, gradient): ∇



When this symbol is used as a sign of omission, it must be spaced accordingly (see §120c). In all other cases, it must be spaced in accordance with the rule above.

(1) $\nabla f(1, 2) = x_i - 3j$



(2) The symbol “ ∇ ” is called “del.”



(3) $||\nabla f(a)||$



(4) $s\nabla t + t\nabla s$



(5) $\nabla(r^n)$




(6) $T \cdot \nabla (r_1 + r_2) = 0$




d. Empty Set (null set, void set):

Represented by Zero With a Slant or Vertical Bar Through It \emptyset or ϕ



Represented by Facing Braces { }



(3) 1^∞



(4) $S_\infty = \frac{a}{1-r}$



(5) $\sum_{n=-\infty}^{\infty} f(n)$



(6) $-\infty < x < \infty$



(7) $n \rightarrow \infty$



(8) $\langle -\infty, +\infty \rangle = \mathbb{R}$

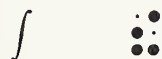


(9) $f'(x) = 0 \text{ or } \infty$

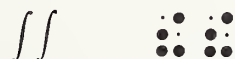


g. Integral:

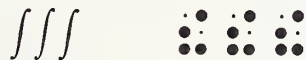
Single Integral



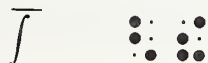
Double Integral



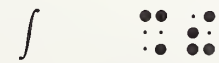
Triple Integral



Upper Integral



Lower Integral



The horizontal bar directly over or under the integral sign must not be treated as a modifier. The symbols for the upper and lower integral signs must be transcribed as shown in the list above. Modifiers other than the horizontal bar must be transcribed according to the five-step rule for the transcription of modified expressions.

(1) $\int (du+dv) = \int du + \int dv$



$$(2) \quad d \int f(x) dx$$



$$(3) \quad V = \iiint dx \, dy \, dz$$



$$(4) \quad \int_a^b f(x) dx = F(x) \Big|_a^b$$



$$(5) \quad \int_1^3 \frac{dx}{x^2 - 2x + 5}$$



$$(6) \quad \int_1^2 \int_0^{x-1} y \, dy \, dx$$



$$(7) \quad \int_a^b f(x) dx = 0$$



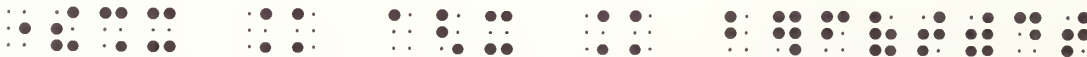
$$(8) \quad \int_a^b f(x) dx = 0$$



$$(9) \quad \int_0^\infty f(x) dx$$



$$(10) \quad \int_{x=a}^{x=b} f(t) dt$$



$$(11) \quad \iint_Q f(x, y) dy \, dx$$



h. Partial Derivative (round d): ∂

$$(1) \quad \frac{\partial f}{\partial x}$$



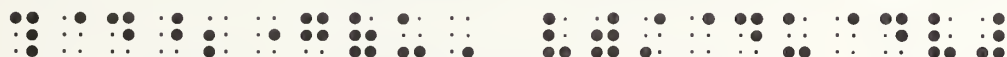
$$(2) \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$$



$$(3) \frac{\partial h}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u}$$



$$(4) \frac{\partial^2 g(u, v)}{\partial u \partial v}$$



$$(5) \partial / \partial y (\partial b / \partial x)$$



i. Quantifiers:

Existential Quantifier

There Exists, for Some

\exists



There Exists Uniquely
for Exactly One

$\exists|$



Universal Quantifier

For All, For Each,
For Every

\forall or \forall



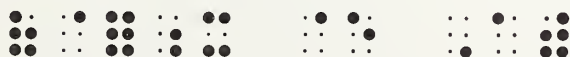
$$(1) \exists x, x < \frac{1}{n}$$



$$(2) (\exists x) (\exists y) [x+y = 85]$$



$$(3) (\exists x \in A)$$



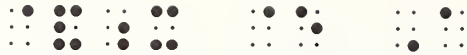
$$(4) \exists|_x$$



$$(5) \exists|_v | v = -v \text{ means "there exists exactly one } v \text{ such that } v \text{ equals } -v."$$



$$(6) \quad \forall x \in A$$



$$(7) \quad (\forall_x)$$



$$(8) \quad V_x V_y = \frac{y-x}{x+y}$$



(9) $\forall x \forall y \neq 0 (x \div y)$



§136. **Spaced Miscellaneous Symbols:** A space must be left before and after the symbols listed below even when these symbols are preceded or followed by a sign of operation. However, no space should be left between these symbols and a sign of grouping, a braille indicator, or a punctuation mark which applies to them.

a. At Sign: @ 

(1) He sold 5 pens @ 45¢ each.



(2) The symbol (@) means “at”.



b. Check Mark: \checkmark 

A space must be left before and after a single check mark. A sequence of two or more check marks must be written unspaced, but the combination as a whole must be preceded and followed by a space.

(1) $\forall 1+2=3$



(2) $-6 = -6 \vee$.



(3) $\forall \forall$ pens



(4) (v) butter, (vv) milk.



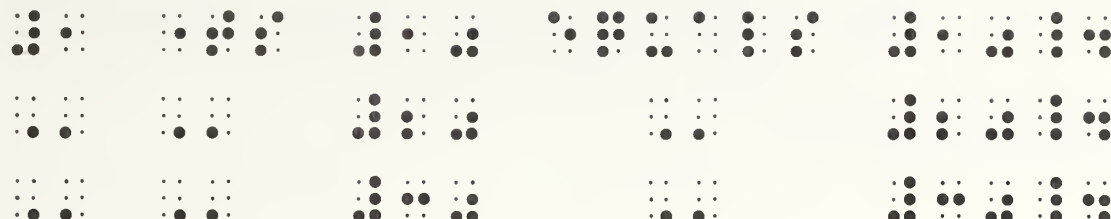
c. Ditto Marks: “ 

A ditto mark must be centered beneath the material to which it applies. It must be separated from any expression which precedes or follows it by at least one space.

(1) 1 times 10 equals 10.

“ “ 20 “ 20.

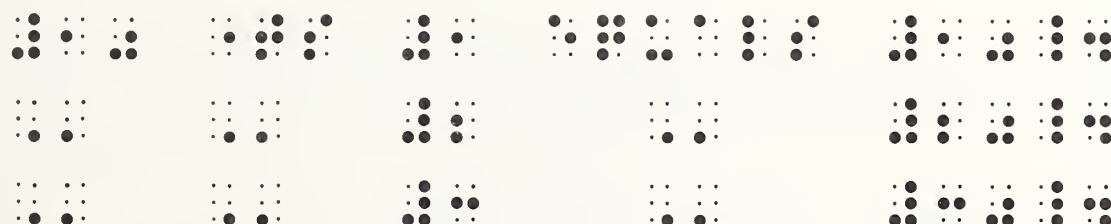
“ “ 30 “ 30.




(2) 10 times 1 equals 10.

“ “ 2 “ 20.

“ “ 3 “ 30.



d. Since (because): 

(1) $\therefore x = y, x^2 = y^2$.



(2) $(\therefore) RS = RT$



e. Therefore:

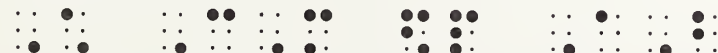
Normal



Negated (it does not follow that)



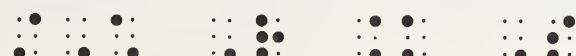
(1) $\therefore CM \perp AB$



(2) \therefore The solution set is $\{\pm 3\}$.



(3) $\nexists R = S$

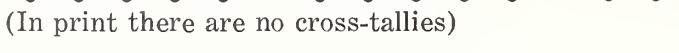









c. Tally Mark: | :●
:●
:●

Tally marks must be grouped in braille just as they are grouped in print. When a group of tally marks is crossed through, the cross-tally must be treated as an additional tally mark which should be added to its group.

One space must be left before and after a group of tally marks even when the group is preceded or followed by a sign of operation. However, when the combination of symbols cannot be accommodated on one braille line, transition to another braille line may take the place of the required space. No space should be left between a group of tallies and a braille indicator, a sign of grouping, or a punctuation mark applying to it.

When a tally mark is followed by punctuation requiring the punctuation indicator, the multipurpose indicator \therefore must be placed between the tally mark and the punctuation indicator.

- (1) 
(In print there are no cross-tallies)
- (2) 
(In print the first two groups of tallies have a cross-tally)
- (3) $22_7 =$ 
- (4) 
- (5) $(||||) + (|||)$ 
- (6) $(||| + |||)$ 
- (7) 
- (8) 

SUPERPOSED SIGNS

§138. **Order of Superposed Signs:** Superposed signs are signs which are placed one upon another so that one sign extends beyond the boundary of the other.

a. Since it may be difficult to distinguish the basic sign from the superposed sign, the following order of preference should be used as a guide. A symbol lower on the list must be regarded as being superposed upon a symbol higher on the list.

- i. Integral sign
- ii. Signs of operation
- iii. Horizontal and vertical bars
- iv. Signs of shape
- v. Signs of comparison
- vi. Signs not listed above

§139. Transcription of Superposed Signs:

a. The components of a superposed sign must be unspaced and transcribed in the following order:

- i. The symbol for the higher sign on the list must be written first.
- ii. The superposition indicator $\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$ must be written next.
- iii. The symbol for the lower sign on the list must be written third.
- iv. The termination indicator $\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$ must be written last.

b. If two signs belong to the same category, the superposition may be represented in either order, provided the same order is followed consistently throughout the text.

(1) \boxminus

$\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

(2) \mathbb{E}

$\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$ or $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

(3) \mathbb{A}

$\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$ or $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

§140. **Integral Modified by Superposition:** The most common integrals with superposed symbols are listed below. Unlisted integrals modified by superposition must be transcribed in accordance with the rules for superposed signs. These signs must be spaced and punctuated as unmodified integral signs.

Integral With Superposed Circle

\oint $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

Integral With Superposed Infinity

\oint $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

Integral With Superposed Rectangle

\oint $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

Integral With Superposed Square

\oint $\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

(1) \oint

$\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

(2) $\oint \frac{dq}{T} < 0$

$\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

(3) $\oint P dx + Q dy$

$\begin{smallmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{smallmatrix}$

$$(4) \oint_3 y^2 dx + x dy$$



$$(5) \oint f(x, y) dy$$



§141. **Signs of Comparison Modified by Superposition:** The most common signs of comparison with superposed signs are listed below. Unlisted signs of comparison modified by superposition must be transcribed in accordance with the rules for superposed signs. They must be spaced and punctuated as any other sign of comparison.

Dot

Between Bars of Equals Sign



Within Inclusion Sign



Within Reverse Inclusion Sign



Equals Sign

Through Inclusion Sign



Through Reverse Inclusion Sign



Greater Than

Nest of Two With Straight Sides
(is large compared with)



Nest of Two With Curved Sides



Horizontal Bar

Through Inclusion Sign



Through Reverse Inclusion Sign



Less Than

Nest of Two With Straight Sides
(is small compared with)



Nest of Two With Curved Sides



Vertical Bar

Through Shaft of Right-Pointing Arrow



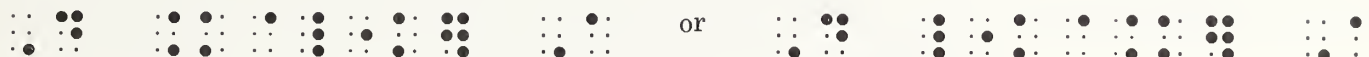
Through Shaft of Left-Pointing Arrow



(1) $\pi \overline{\cdot} 3.14$



(2) $D \Subset A$



(3) $r \gg a$



(4) $T \supset u$



(5) $l \ll n$



(6) $A \dashrightarrow B$



§142. Signs of Shape Modified by Superposition: A shape modified by superposition is a basic sign having a superposed sign which extends beyond the boundary of the basic shape. Such signs are transcribed in accordance with the rules for superposed signs in §139. Signs of shape modified by superposition must be spaced and punctuated as other signs of shape.

(1) Describe $\star A$.



(2) Divide the right half of \boxplus into thirds.



HOMEWORK

Prepare the following homework for submission to your teacher. Proofread carefully.

EXERCISE 12

1. In dividing .489 by .3, you must change the divisor to .3_^. Select the correct form of the dividend.

- a. .4_^89 b. .48_^9 c. .489_^

2. $1/x$ approaches ? as x tends to either $+\infty$ or $-\infty$.
3. Let g and g^* be harmonic in Ω , and let $\frac{\partial g}{\partial n} = \frac{\partial g^*}{\partial n}$ on $\partial \Omega$. Show that $g^* = g + K$, where K is constant.
4. Let $\nabla \cdot F = 0$ in a convex region Ω . Show that F can be expressed there in the form $F = \nabla \times V$, where $\nabla \cdot V = 0$ and $\nabla^2 V = -\nabla \times F$.
5. Show that $(R \cap S) \cap \emptyset = R \cap (S \cap \emptyset)$ if $R = \{1, 3, 5, 7\}$ and $S = \{1, 2, 3, 4\}$.
6. Is the following statement true or false?
 $\{ \} \subset \{1, 2\}$

Compute the indicated number.

7. $7!$
8. $\frac{10!}{8!}$
9. $\frac{6!}{3!2!}$
10. $\frac{10!}{5!5!}$
11. Simplify $\frac{(x-y)!}{(x-y-1)!}$ given x and y are positive integers with $x > y$ and all denominators are meaningful.

12. Find a 1-form ω , for which $d\omega = (x^2 + y^2) dx dy$, and use this to evaluate

$$\iint_D (x^2 + y^2) dx dy$$

where D is the region inside the square $|x| + |y| = 4$ and outside the circle $x^2 + y^2 = 1$.

13. Show that the area of a region D to which Green's theorem applies may be given by

$$A(D) = \frac{1}{2} \int_{\partial D} (x dy - y dx).$$

Read each of the following statements and tell whether the statement is true. Assume that the domain of each variable is \mathbb{R} .

14. $(\forall_s)(s \cdot 0 = 0)$
15. $(\exists_a)(a+1 \neq 3)$
16. $(\exists_r)(r-2 = 0)$
17. $(\forall_x)(\exists_y)(x+y \neq y+x)$

Name the coins that might be used in making change from a half-dollar.

18. A basket of grapes @ 20ϕ .
19. A can of peaches @ 25ϕ .
20. Find the sum of $2\frac{1}{2}$ inches and 4 inches. Answer: $2+4 = 6$, $6+\frac{1}{2} = 6\frac{1}{2}$.
21. Answer the following questions.

What part of 24 is 14?

“ “ “ 144 “ 48?

“ “ “ 30 “ 9?

22. $\because 3x+2y = 8$, if we substitute 0 for x we obtain $3(0)+2y = 8$. $\therefore y = 4$.

THEOREM 23. Let f be continuous on the interval $[a, b]$, and let $f'(x)$ exist for $a < x < b$. If, in addition, $f(a) = f(b)$ then there is a point x_0 with $a < x_0 < b$ at which $f'(x_0) = 0$.

If f is a constant function, any choice of the point x_0 will do. If f is not a constant, then it must have either an interior minimum or an interior maximum at some point x_0 on the open interval $a < x < b$; and since f is differentiable there, $f'(x_0) = 0$. †

24. In optics we use the micron, the millimicron (m_μ), and the Angstrom (\AA) as units of wavelength. The center of the visible region can be expressed as 0.555 micron, 555 m_μ , or 5550 \AA .

25. Is it conceivable that electromagnetic theory might some day be able to predict the value of $c(3 \times 10^8 \text{ meters/sec})$, not in terms of ϵ_0 and μ_0 but directly and numerically without recourse to any measurements?

26. In the mks system S is expressed in (watts)/(meter)². Does the direction of S give the direction in which the energy moves?

27. If the marks ||||| equal the numeral 11_{six}, what numeral does ||||| equal in base six?

28. The boys marked the number of fish they caught in this way:

|||||

How many fish did they catch?

29. Name each \angle in rectangle QRST.

30. In Gauss's law,

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q,$$

is \mathbf{E} the electric field intensity attributable to the charge?

31. In the figure given, assume that both charges are positive. Show that E at point p in that figure, assuming $r \gg a$, is given by

$$E = \frac{1}{4\pi\epsilon_0} \frac{2q}{r^2}.$$

LESSON 13

FUNCTION NAMES AND THEIR ABBREVIATIONS

§143. **Function Names:** The most common function names and their abbreviations are listed below. Abbreviated function names must not be considered ordinary abbreviations. They are mathematical expressions and must be transcribed in accordance with the principles defined below. Unlisted function names or their abbreviations must be transcribed as shown in print and are subject to the same rules.

ABBREVIATED FUNCTION NAME	UNABBREVIATED FUNCTION NAME	BRAILLE EQUIVALENT
amp	amplitude	$\cdot \cdot \cdot \cdot \cdot \cdot$
antilog	antilogarithm	$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$
arc (no abbreviation)	arc	$\cdot \cdot \cdot \cdot \cdot \cdot$
arg	argument	$\cdot \cdot \cdot \cdot \cdot \cdot$
colog	cologarithm	$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$
cos	cosine	$\cdot \cdot \cdot \cdot \cdot \cdot$
cosh	hyperbolic cosine	$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$
cot	cotangent	$\cdot \cdot \cdot \cdot \cdot \cdot$
coth	hyperbolic cotangent	$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$
covers	coversine	$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$
csc	cosecant	$\cdot \cdot \cdot \cdot \cdot \cdot$
csch	hyperbolic cosecant	$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$
ctn	cotangent	$\cdot \cdot \cdot \cdot \cdot \cdot$
ctnh	hyperbolic cotangent	$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$
det	determinant	$\cdot \cdot \cdot \cdot \cdot \cdot$
erf	error function	$\cdot \cdot \cdot \cdot \cdot \cdot$
exp	exponential	$\cdot \cdot \cdot \cdot \cdot \cdot$
exsec	exsecant	$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$

ABBREVIATED FUNCTION NAME	UNABBREVIATED FUNCTION NAME	BRAILLE EQUIVALENT
grad	gradient	$\begin{smallmatrix} \bullet\bullet & \bullet\cdot & \cdot\cdot & \bullet\bullet \\ \cdot\cdot & \bullet\cdot & \cdot\cdot & \cdot\cdot \end{smallmatrix}$
hav	haversine	$\begin{smallmatrix} \bullet\cdot & \cdot\cdot & \bullet\cdot \\ \bullet\cdot & \cdot\cdot & \bullet\cdot \end{smallmatrix}$
im	imaginary part	$\begin{smallmatrix} \cdot\cdot & \bullet\bullet \\ \bullet\cdot & \cdot\cdot \end{smallmatrix}$
inf	infimum	$\begin{smallmatrix} \cdot\cdot & \bullet\bullet & \bullet\bullet \\ \cdot\cdot & \bullet\cdot & \cdot\cdot \end{smallmatrix}$
lim	limit	$\begin{smallmatrix} \bullet\cdot & \cdot\cdot & \bullet\bullet \\ \bullet\cdot & \cdot\cdot & \bullet\cdot \end{smallmatrix}$
$\overline{\lim}$ or $\overline{\lim}$	upper limit	$\begin{smallmatrix} \bullet\cdot & \bullet\cdot & \bullet\cdot & \bullet\bullet \\ \bullet\cdot & \bullet\cdot & \cdot\cdot & \bullet\cdot \end{smallmatrix}$
$\underline{\lim}$ or $\underline{\lim}$	lower limit	$\begin{smallmatrix} \bullet\bullet & \bullet\cdot & \cdot\cdot & \bullet\bullet \\ \cdot\cdot & \bullet\cdot & \cdot\cdot & \cdot\cdot \end{smallmatrix}$
ln	natural logarithm	$\begin{smallmatrix} \bullet\cdot & \bullet\bullet \\ \bullet\cdot & \cdot\cdot \end{smallmatrix}$
log	logarithm	$\begin{smallmatrix} \bullet\cdot & \cdot\cdot & \bullet\bullet \\ \bullet\cdot & \bullet\cdot & \cdot\cdot \end{smallmatrix}$
max	maximum	$\begin{smallmatrix} \bullet\bullet & \cdot\cdot & \bullet\bullet \\ \cdot\cdot & \cdot\cdot & \bullet\bullet \end{smallmatrix}$
min	minimum	$\begin{smallmatrix} \bullet\bullet & \cdot\cdot & \bullet\bullet \\ \cdot\cdot & \bullet\cdot & \bullet\cdot \end{smallmatrix}$
mod	modulo	$\begin{smallmatrix} \bullet\bullet & \bullet\cdot & \bullet\bullet \\ \cdot\cdot & \bullet\cdot & \cdot\cdot \end{smallmatrix}$
re	real part	$\begin{smallmatrix} \bullet\cdot & \cdot\cdot \\ \bullet\cdot & \cdot\cdot \end{smallmatrix}$
sec	secant	$\begin{smallmatrix} \cdot\cdot & \bullet\cdot & \bullet\bullet \\ \bullet\cdot & \bullet\cdot & \cdot\cdot \end{smallmatrix}$
sech	hyperbolic secant	$\begin{smallmatrix} \cdot\cdot & \bullet\cdot & \bullet\bullet & \bullet\cdot \\ \bullet\cdot & \bullet\cdot & \cdot\cdot & \bullet\cdot \end{smallmatrix}$
sin	sine	$\begin{smallmatrix} \cdot\cdot & \cdot\cdot & \bullet\bullet \\ \bullet\cdot & \cdot\cdot & \bullet\cdot \end{smallmatrix}$
sinh	hyperbolic sine	$\begin{smallmatrix} \cdot\cdot & \bullet\cdot & \bullet\bullet & \bullet\cdot \\ \bullet\cdot & \cdot\cdot & \bullet\cdot & \bullet\cdot \end{smallmatrix}$
sup	supremum	$\begin{smallmatrix} \cdot\cdot & \cdot\cdot & \bullet\bullet \\ \bullet\cdot & \bullet\cdot & \bullet\cdot \end{smallmatrix}$
tan	tangent	$\begin{smallmatrix} \cdot\cdot & \cdot\cdot & \bullet\bullet \\ \bullet\cdot & \cdot\cdot & \bullet\cdot \end{smallmatrix}$
tanh	hyperbolic tangent	$\begin{smallmatrix} \cdot\cdot & \cdot\cdot & \bullet\bullet & \bullet\cdot \\ \bullet\cdot & \cdot\cdot & \bullet\cdot & \bullet\cdot \end{smallmatrix}$
vers	versine	$\begin{smallmatrix} \bullet\cdot & \cdot\cdot & \bullet\bullet & \bullet\cdot \\ \bullet\cdot & \cdot\cdot & \bullet\cdot & \cdot\cdot \end{smallmatrix}$

§144. Contractions With Function Names: Contractions must not be used in any abbreviated function name. Although contractions may be used with unabbreviated function names, an unabbreviated function name must not be contracted when used in conjunction with an abbreviated function name or with related mathematical symbols.

§145. Spacing and Nonuse of the English Letter Indicator With Function Names and Their Abbreviations:

a. A space must be left after a function name or its abbreviation. A space must be left even when the function name or its abbreviation is directly followed by a sign of operation. The expression following the function name or its abbreviation must be spaced in accordance with its appropriate spacing rules.

No space should be left before a function name or its abbreviation unless it is preceded by a symbol requiring a space.

The English letter indicator must not be used with an English letter, a short-form combination, or a Roman numeral in regular type following a function name or its abbreviation.

(1) $\sin x$

(2) arc ab

(3) Sine I

$$(4) \quad \cos \theta = \frac{1}{\sin \theta}$$

(5) $\text{ctn } \neg A = \neg \text{ctn } A$

(6) $1/\cos - \cos = \tan \cdot \sin$

$$(7) \quad \frac{1}{\cos} \cdot \cos = \tan \cdot \sin$$

(8) $\sin (\theta+90^{\circ})=\cos \theta$

(9) $y = 3 \tan 2x$

The dot patterns are as follows:

- Pattern 1: All dots are filled.
- Pattern 2: The top-left dot is empty; all other dots are filled.
- Pattern 3: The top-right dot is empty; all other dots are filled.
- Pattern 4: The bottom-left dot is empty; all other dots are filled.
- Pattern 5: The bottom-right dot is empty; all other dots are filled.
- Pattern 6: The top-left and bottom-right dots are empty; all other dots are filled.
- Pattern 7: The top-right and bottom-left dots are empty; all other dots are filled.
- Pattern 8: The top-left and top-right dots are empty; all other dots are filled.
- Pattern 9: The top-right and bottom-right dots are empty; all other dots are filled.
- Pattern 10: The top-left and bottom-right dots are empty; all other dots are filled.
- Pattern 11: The top-right and bottom-left dots are empty; all other dots are filled.
- Pattern 12: The top-left, top-right, and bottom-right dots are empty; all other dots are filled.

$$(10) \quad a^2 = 2ac \cos \beta + c^2$$

$$(11) \quad y = 2\sin x + \sin 2x$$

(12) $2\sin x + 3\cos y$

(13) $\sin x - \sin y$

$$(14) \quad \sin (\alpha+\beta)+\sin (\alpha-\beta)$$

$$(15) \quad y' = x \cos \phi - y \sin \phi$$

$$(16) \quad 2\sqrt{x} \sin \sqrt{x} + 2\cos \sqrt{x} = C$$

$$(17) \quad a \sin \frac{x}{a} \cdot \frac{1}{a} = \sin \frac{x}{a}$$

$$(18) \quad y = \sqrt{\cot x}$$

$$(19) \quad r[3\cos \theta + 4\sin \theta] = 5$$

$$(20) \quad 7(\cos 20^\circ + i \sin 20^\circ)$$

$$(21) \quad \sin 2\alpha = 2 \sin\alpha$$

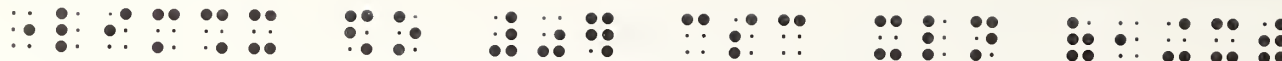
$$(22) \quad 6\sin 2A \cos 4A$$

Figure 1 shows three 3x3 dot patterns. Pattern (a) has 10 dots: (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), and (2,2) is also present. Pattern (b) has 11 dots: (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (2,2), and (2,2) is also present. Pattern (c) has 12 dots: (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (2,2), (2,2), and (2,2) is also present.

$$(23) \quad \cos 203^\circ \csc 203^\circ$$

$$(24) \quad \{\sin x \mid \sin x + 2 < +1\}$$

$$(7) \lim_{x \rightarrow 0} \csc x \ln(1+x)$$



$$(8) \lim_{\theta \rightarrow \theta_0} (\tan \theta)$$



§147. Superscripts and Subscripts With Function Names:

a. When a function name or its abbreviation carries a superscript or subscript, the required space after the function name must follow the superscript or subscript. A letter, numeral, or other mathematical expression following this space assumes the same level as the function name.

The subscript indicator must not be used when an abbreviated function name carries a numeric subscript only on the first level below the baseline of writing. However, the subscript indicator must be used if an unabbreviated function name carries a numeric subscript.

$$(1) \sin^2 x$$



$$(2) \text{sine}^2 x$$



$$(3) \sin^2 A + \cos^2 (B+A)$$



$$(4) \cot^{-1} x + \frac{\pi}{2} - \tan^{-1} x$$



$$(5) \sin^2 90^\circ + \cos^2 90^\circ = 1$$



$$(6) (1 - \sin^2 x)^2 \cos^2 x$$



$$(7) \sin^2 \theta \times \frac{\cos^2 \theta}{\sin^2 \theta} - 1$$



$$(8) \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$$



$$(9) \log_n .125 = -.6$$



(8) $3^{\log_3 9}$



(9) $e^{\sin x} + e^{\sin y}$



(10) $3^{\log_3 7} + 2^{\log_2 5}$



(11) $2^{\sec x} = y$



(12) $a^{\log_a x} = x$



(13) $e^{\sin x} = a > y$



§148. Enclosed Lists With Abbreviated or Unabbreviated Function Names: Although the numeric indicator must not be used at the beginning of an item in an enclosed list, it must be used before a numeral or decimal point and a numeral in regular type following a function name or its abbreviation.

(1) $(2\sin 30^\circ, 3\cos 60^\circ)$



(2) $(\cos .8000, 2\cos .8000)$



§149. Punctuation With Function Names: An abbreviated function name is a mathematical expression and must be punctuated accordingly. An unabbreviated function name must be punctuated in the mathematical or literary mode according to its context.

(1) Some trigonometric functions are sin, cos, and tan.



(2) "Arcsin" is the "inverse sine."



(3) What is the meaning of logsine?



-

-

§150. General Principles: Unless absolutely unavoidable, a mathematical expression must not be divided between braille lines or braille pages. In order to avoid a division, if there is insufficient space on the remainder of a line to accommodate the expression, the space must be left blank, and the entire expression must be brought down to the next line.

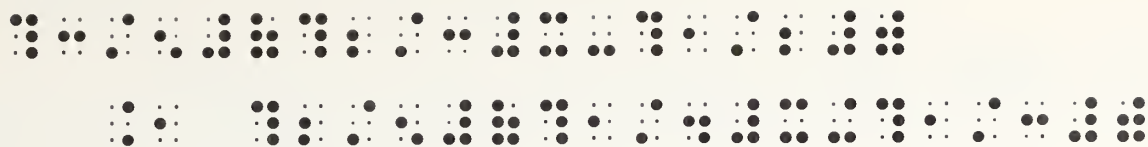
§151. Division of Long Numerals: A long numeral which cannot be contained on one braille line within the margin in effect must be divided after a comma if a comma is present, and a hyphen must be inserted. If the numeral does not contain a comma, the hyphen may be inserted after any digit.

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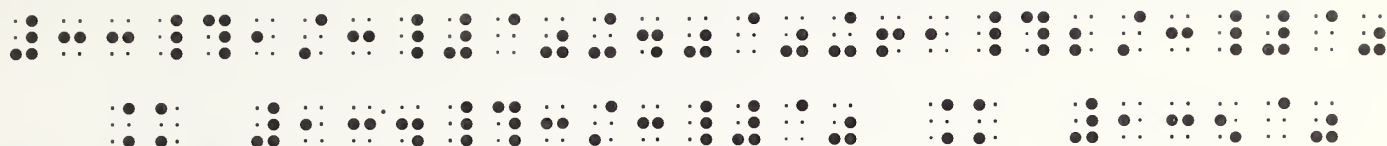
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$$(3) \quad \frac{3}{5} \left(\frac{2}{3}x - \frac{1}{2} \right) > \frac{2}{5} \left(\frac{1}{4}x + \frac{1}{3} \right)$$

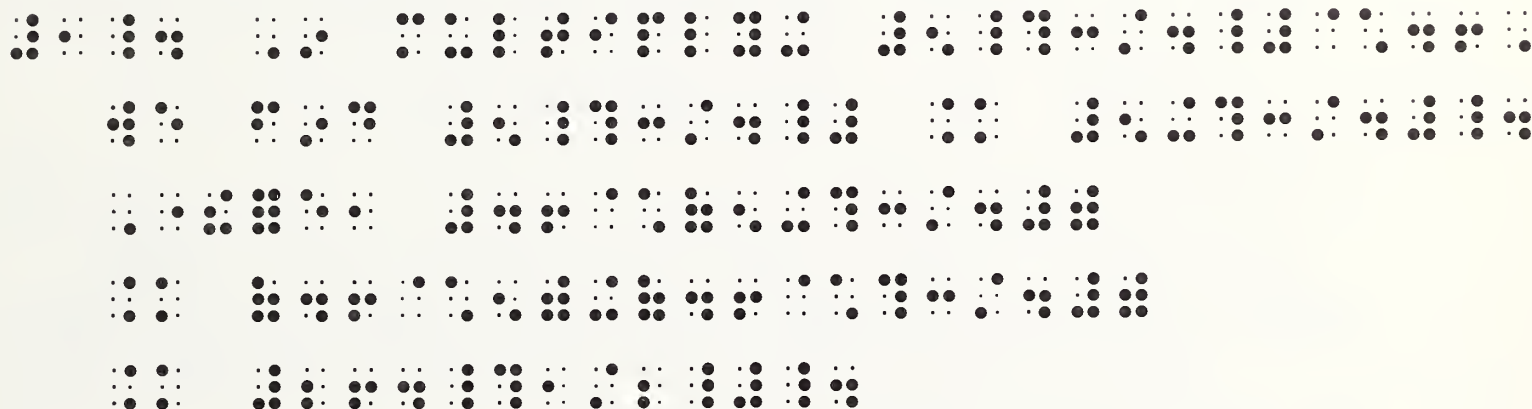


$$(4) \quad 33\frac{1}{3}\% + 40\% + 61\frac{2}{3}\% = 134\frac{3}{3}\% = 135\%$$



$$(5) \quad 1. \text{ In multiplying } 5\frac{3}{4} \times 46, \text{ we find } 5\frac{3}{4} = 5 + \frac{3}{4}.$$

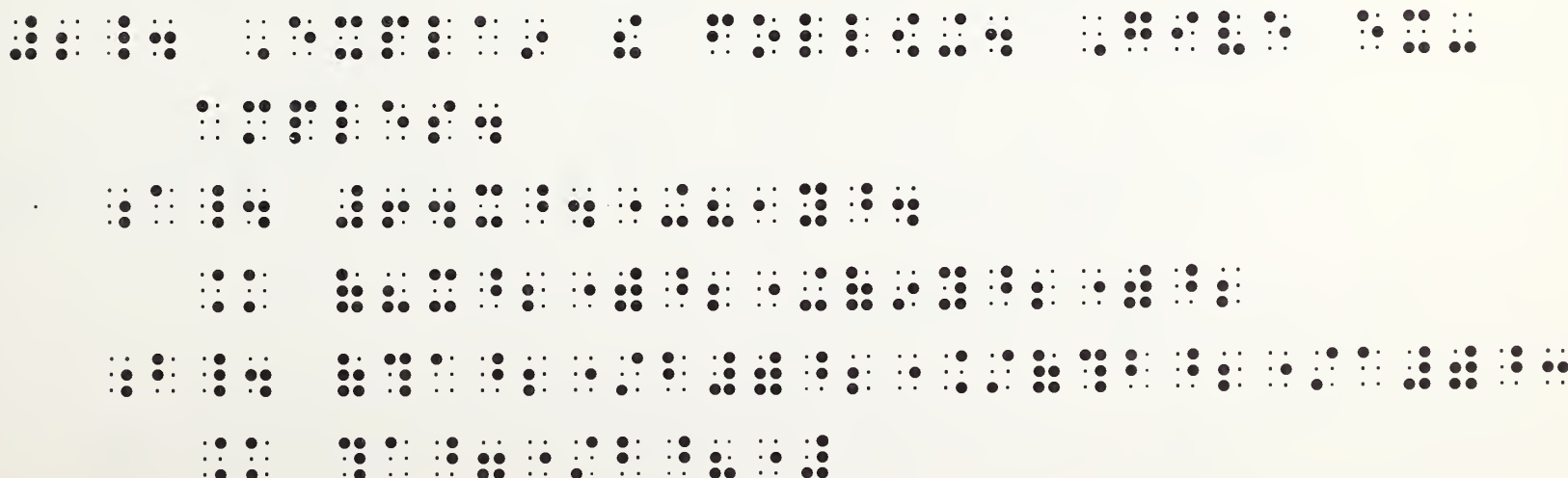
$$\text{Therefore, } 46 \times \left(5 + \frac{3}{4} \right) = (46 \times 5) + \left(46 \times \frac{3}{4} \right) = 264\frac{1}{2}.$$



(6) 2. Explain the following. Give examples.

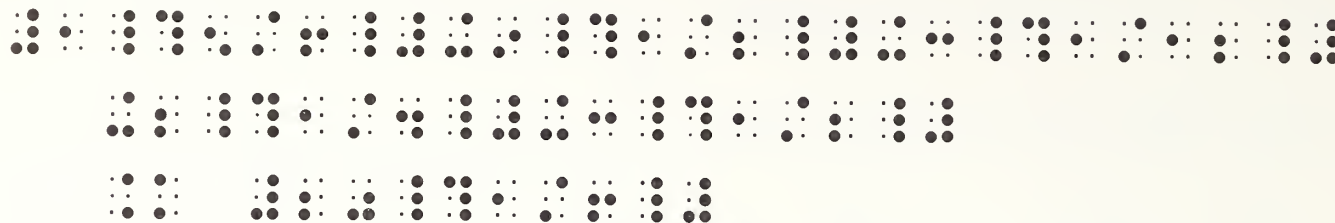
$$a. \quad 64x^4 + 81y^4 = (8x^2)^2 + (9y^2)^2$$

$$b. \quad \left(\frac{a^2}{b} \right)^2 \div \left(\frac{b^2}{a} \right)^3 = \frac{a^7}{b^8}$$

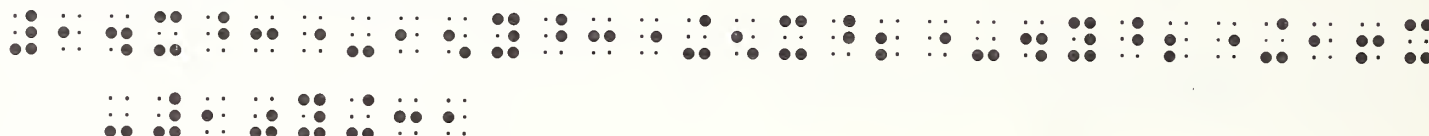


iii. A division may be made before a sign of operation on the baseline of writing. If the transition to a new line is made before a minus sign, the numeric indicator must be used after the minus sign when followed by a numeral or a decimal point and a numeral.

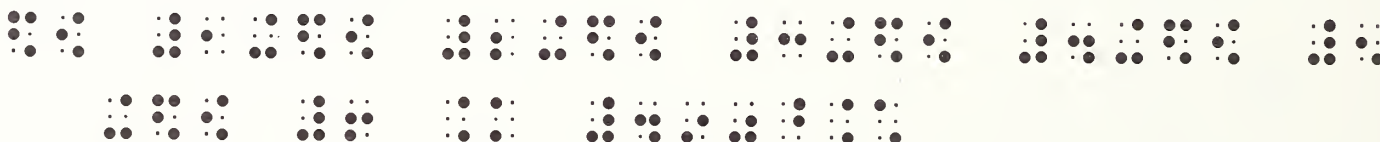
$$(1) \quad 1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2} = 20\frac{1}{6}$$



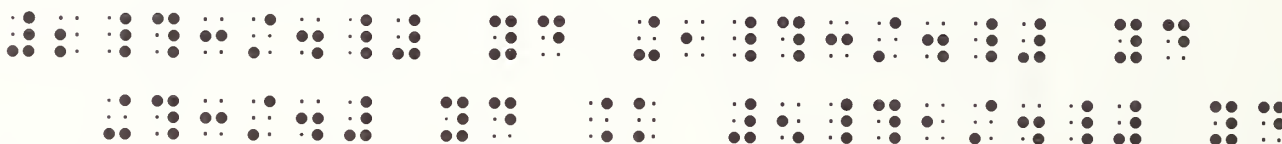
$$(2) \quad 14x^3 - 15y^3 + 5x^2 - 4y^2 + 16x - 10y + 31$$



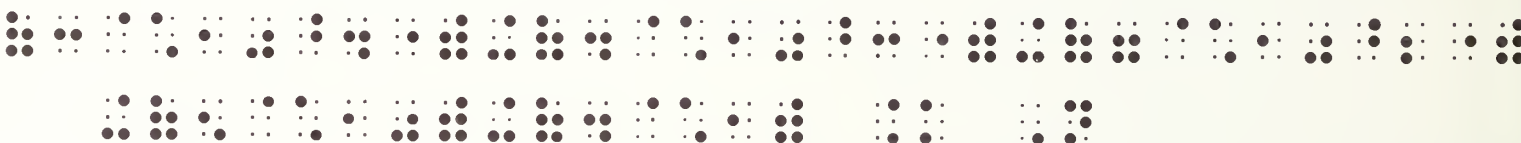
$$(3) \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 490^\circ$$



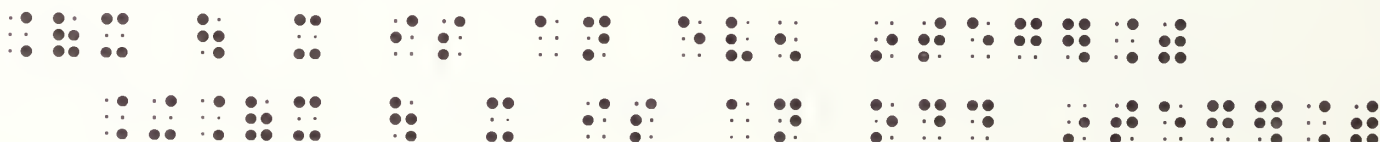
$$(4) \quad 2\frac{3}{4} \text{ yd} + 1\frac{3}{4} \text{ yd} + \frac{3}{4} \text{ yd} = 5\frac{1}{4} \text{ yd}$$



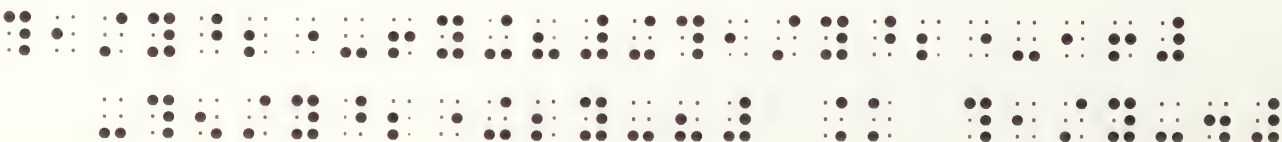
$$(5) \quad (3 \times 10^4) + (4 \times 10^3) + (7 \times 10^2) + (5 \times 10) + (4 \times 1) = N$$



$$(6) \quad \{x \mid x \text{ is an even integer}\} \cup \{x \mid x \text{ is an odd integer}\}$$



$$(7) \quad \frac{1}{y^2 - 6y + 8} + \frac{1}{y^2 - 16} - \frac{5}{y^2 + 2y - 8} = \frac{1}{y - 4}$$



v. A division may be made before the baseline indicator. Thus, if a baseline indicator is required to show a return to the baseline of writing after a superscript or subscript, it must be the first symbol on the new braille line. However, if transition to a new line is made after a numeric subscript not requiring the subscript indicator, the baseline indicator is not required before the runover on the new line.

(1) $(49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2) \div 7x^3y^2$

Braille representation of equation (1). The expression is written across two lines. The first line contains the polynomial $(49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2)$ followed by a division symbol and the divisor $7x^3y^2$. The second line contains the result of the division, which is a polynomial with terms of degree 4 in x and degree 4 in y.

(2) $(16x^4 + 8x^3y + 4x^2y^2 + 2xy^3 + y^4)(2x - y) = 32x^5 - y^5$

Braille representation of equation (2). The expression is written across two lines. The first line contains the product of two polynomials: $(16x^4 + 8x^3y + 4x^2y^2 + 2xy^3 + y^4)(2x - y)$. The second line contains the result $32x^5 - y^5$.

(3) $144 ft^2 + 112 ft^2 + 145 ft^2 - 131 ft^2 = N$

Braille representation of equation (3). The expression is written across two lines. The first line contains the sum and difference of terms: $144 ft^2 + 112 ft^2 + 145 ft^2 - 131 ft^2$. The second line contains the result N .

(4) $\frac{\sqrt{0.4523} \times (2.6275)^3}{43690 \times 0.000982}$

Braille representation of equation (4). The expression is written across two lines. The first line contains the numerator: $\sqrt{0.4523} \times (2.6275)^3$. The second line contains the denominator: 43690×0.000982 .

(5) $\pm \left(\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}}{3} \right)^{\frac{3}{2}}$

Braille representation of equation (5). The expression is written across two lines. The first line contains the base of the power: $\pm \left(\frac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}}{3} \right)$. The second line contains the exponent $\frac{3}{2}$.

(6) $(\pm) a_{1i_1} a_{2i_2} a_{3i_3} a_{4i_4} \dots a_{ni_n}$

Braille representation of equation (6). The expression is written across two lines. The first line contains the product of terms: $(\pm) a_{1i_1} a_{2i_2} a_{3i_3} a_{4i_4} \dots a_{ni_n}$. The second line contains the result of the product.

(7) $(((((Z_2)_3)_5)_7)_{11})_{15})_{17}$

Braille representation of equation (7). The expression is written across two lines. The first line contains the nested parentheses: $(((((Z_2)_3)_5)_7)_{11})_{15})_{17}$. The second line contains the result of the nested parentheses.

(8) $a_1 b_2 c_3 + a_3 b_1 c_2 + a_2 b_3 c_1 - a_3 b_2 c_1 - a_1 b_3 c_2 - a_2 b_1 c_3$

Braille representation of equation (8). The expression is written across two lines. The first line contains the sum and difference of terms: $a_1 b_2 c_3 + a_3 b_1 c_2 + a_2 b_3 c_1 - a_3 b_2 c_1 - a_1 b_3 c_2 - a_2 b_1 c_3$. The second line contains the result of the sum and difference.

vi. A division may be made before a superscript or subscript change-of-level indicator, or before a sign of comparison, a sign of operation, or a fraction line within a superscript or subscript.

When an expression is divided before a superscript or subscript change-of-level indicator, the indicator must be placed before the continuation of the expression on the new braille line.

If transition to a new braille line is made within a superscript or subscript, the level in effect is not changed when the division is made before a sign of operation or a fraction line. However, if the transition is made before a sign of comparison, the level in effect is terminated, and the level must be restated before the sign of comparison on the new braille line.

$$(1) \quad N_{wxyz\dots}^{abcd\dots}$$

[illegible]

$$(2) \quad x^1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots + \frac{1}{n}$$

$$(3) \quad \frac{1}{2} \left[\frac{1}{2} \sin 2u - u \right] \frac{P(u=0)}{A(u=0)}$$

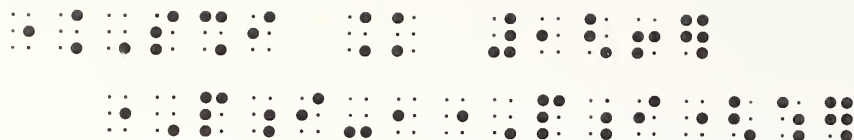
vii. A division may be made between factors enclosed in signs of grouping.

$$(1) \quad (3x^2y)(3x^2y)(3x^2y)(3x^2y) = 81x^4y^4$$

viii. A division may be made after the termination indicator of a modified expression or a radical. The five components of a modified expression must not be separated from each other by transition to a new braille line unless the modified expression as a whole is so long that it cannot fit inside the margins in effect. In such cases, a division should be made in the order of preference listed above.

$$(1) \quad \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p X_{ijk} = \sum_{k=1}^p \sum_{j=1}^n \sum_{i=1}^m X_{ijk}$$

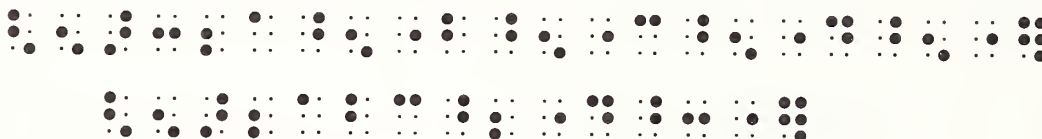
$$(2) \sum_{i=1}^6 \overline{P_i - 1 P_i}$$



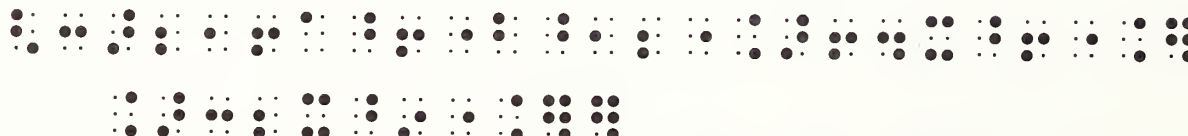
$$(3) \sqrt[3]{x} \sqrt[6]{x^2} \sqrt[12]{x^3} \sqrt[24]{x^4} \sqrt[48]{x^5} \dots$$



$$(4) \sqrt[5]{32a^5b^5c^5d^5} \sqrt[5]{2abc^2d^3}$$



$$(5) \sqrt[3]{216a^6b^{12}} \sqrt{64x^6} \sqrt{32x^9}$$



§153. Nondivision of Symbols and Expressions:

a. The components of the following signs must not be divided between braille lines:

- i. Plus or minus
- ii. Minus or plus
- iii. Plus followed by minus
- iv. Minus followed by plus
- v. Signs of comparison compounded vertically or horizontally
- vi. Superposed signs
- vii. Tally marks belonging to the same group
- viii. Signs of shape with structural or interior modification

b. The following expressions must not be divided between braille lines:

- i. A hyphenated expression containing one or more mathematical components
- ii. An abbreviation and its related preceding or following numeral or letter
- iii. A sign of shape and its identifying numeral, letter, or sequence of letters
- iv. A function name or its abbreviation and the sign following it

HOMEWORK

Prepare the following homework for submission to your teacher. Proofread carefully.

EXERCISE 13

Find the value of each of the following.

1. $\sin \frac{5\pi}{12}$

2. $\tan \frac{7\pi}{12}$

3. $\cot 75^\circ$

4. $\csc 75^\circ$

5. $\text{Sine } 195^\circ$

Prove each of the following reduction formulas.

6. $\tan (90^\circ + \theta) = -\cot \theta$

7. $\sin \left(\frac{3\pi}{2} + x \right) = -\cos x$

Verify that each of the following statements is true.

8. $2 \cos^2 330^\circ - 1 = \cos 660^\circ$

9. $\cos 225^\circ = -\sqrt{\frac{1 + \cos 450^\circ}{2}}$

10. $2 \sin^2 \frac{4\pi}{3} = 1 - \cos \frac{2\pi}{3}$

Prove each of the following identities.

11. $\cot 2\theta = \frac{\cot^2 \theta - 1}{2 \cot \theta}$

12. $\csc 2\theta = \frac{\sec \theta \csc \theta}{2}$

13. $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

Give the value of each of the following numbers.

14. $\text{Arcsin } \frac{1}{2}$

15. $\text{Arc cot } \left(-\frac{1}{\sqrt{3}} \right)$

16. $\text{Arc csc } (-\sqrt{2})$

17. Evaluate $\sin \arccos \frac{3}{5}$ and $\tan \arcsin \left(-\frac{15}{17}\right)$.

Write each logarithmic statement in exponential form.

18. $\log_3 9 = 2$

19. $\log_{\frac{1}{2}} 8 = -3$

20. $\logarithm_7 7 = 1$

21. Is the following statement true? If not, replace the right-hand member with a numeral that makes it true.

$$4^{2\log_4 3} + 3^{2\log_3 4} = 1.$$

Write each expression in simpler form.

22. $e^{\log_e e}$

24. $\exp_8 (\log_4 2)$

23. $\exp_e (\log_{e^2} 3)$

25. $16^{\log_4 2}$

26. Find $\lim_{x \rightarrow 0.6} 2^{25x^2 - 10x - 1}$.

27. Formulate a precise definition for $\lim_{x \downarrow -\infty} f(x) = L$.

28. If $\overline{\lim}_{n \rightarrow \infty} a_n = A$ and $\overline{\lim}_{n \rightarrow \infty} b_n = B$, must it be true that

$$\overline{\lim}_{n \rightarrow \infty} (a_n + b_n) = A + B?$$

29. Find $\overline{\lim}_{n \rightarrow \infty} a_n$ and $\lim_{n \rightarrow \infty} a_n$ when $a_n = (-1)_n$.

If θ is the angle from the first of the given vectors to the second, determine $\sin \theta$ and $\cos \theta$.

30. $(\cos 200^\circ, \sin 200^\circ); (3\cos 100^\circ, 3\sin 100^\circ)$.

31. $(3\cos 20^\circ, 3\sin 20^\circ); (6\cos 350^\circ, 6\sin 350^\circ)$.

32. State the general rules for determining a sine, a cosine, and a tangent. Of a cot, a csc, and a sec.

33. State the name of the following numeral in exponential form.

$$750,000,000,000,000,000,000,000,000.$$

34. Divide the following numeral by the use of the comma. Does this help you read it?

$$756928742059634291852089115762017215762.$$

35. Copy and complete so that the set is a set of equivalent fractions.

$$\left\{ \frac{6}{9}, \frac{?}{18}, \frac{?}{27}, \frac{?}{36}, \frac{?}{63}, \frac{?}{90}, \dots \right\}.$$

36. Does $\{-4, -\pi, -3, -\frac{7}{3}, -\sqrt{2}, -2, -1, -0.5, 0, 0.5, 1, \sqrt{2}, 2, \frac{7}{3}, 3, \pi, 4\}$ represent a set of real numbers?

Determine which of the following statements is true. Give a reason for each answer.

37. $\{[(4 \times 3) - 4] \div 3\} \times 6 \geq (4 \times 3) - [(4 \div 3) \times 6]$

38. $9 \left[\left(\frac{11-4}{3 \cdot 15} - \frac{1}{45} \right) - \frac{1}{45} \right] = 4 \div 4$

39. To factor the expression $-2ab + a^2 + b^2$,

Gail wrote: $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b-a)^2$

Karen wrote: $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a-b)^2$

The teacher called both answers correct. Explain.

40. Find a single numeral to replace N.

$$(9 \times 10^3) + (7 \times 10) + (8 \times \frac{1}{10}) + (5 \times \frac{1}{10^2}) = N$$

Prove each statement. Let $f = \{(x, y) : y = a_0 x^n + a_1 x^{n-1} + \dots + a_{n-2} x^2 + a_{n-1} x + a_n\}$.

41. $f''(x) = n(n-1)a_0 x^{n-2} + (n-1)(n-2)a_1 x^{n-3} + \dots + 2 \cdot 1 a_{n-2}$.

42. $\frac{f''(c)}{2!}$ is the coefficient of $(x-c)^2$ in the expansion of $f(x)$ in powers of $x-c$.

Compute.

43. $\frac{(3.00 \times 10^{15})(1.53 \times 10^{-11})}{2.50(10^6)}$

44. $\frac{(1.21 \times 10^4)(6.937 \times 10^8)}{3.75(10^2)}$

Simplify.

45. $[3t^2 + (-5)t^3 + 2t + -4] + [-3 + (-2)t + 5t^3 + (-7)t^2]$

46. $[(-1)s^4 + 7s^3 + (-3)s^2 + 2s^1 + -10] + [(-3)s^4 + 5s^2 + 11]$

47. What is the sum of the exponential values in

$$s^{1+(-1)+\frac{1}{2}+(-\frac{1}{2})+\frac{1}{4}+(-\frac{1}{4})+\frac{1}{8}+(-\frac{1}{8})?$$

48. Multiply the factors: $(4x^9y)(7x^8y^2)(5x^7y^3)(6x^6y^4)$

$$(2x^5y^5)(x^4y^6)(9x^3y^7)(3x^2y^8)(8xy^9).$$

49. $\lim_{\substack{n \rightarrow \infty \\ m \rightarrow \infty}} \sum_{i=0}^n \sum_{j=0}^m a_{ij} = \lim_{n \rightarrow \infty} \sum_{i=0}^n a_i \sum_{j=0}^m a_i \lim_{m \rightarrow \infty} \sum_{j=0}^m b_j = AB$

50. How do you add 7 ft 2 in and 9 ft 5 in?

51. If a piece of tape is measured with a 6-inch ruler, how many times must you use the ruler if the tape is 108 inches long?

LESSON 14

CONTRACTIONS AND SHORT-FORM WORDS

§154. Review of Contractions and Short-Form Words: In the absence of specific restrictions on the treatment of contractions and short-form words, English braille rules are applied.

The rules for contractions and short-form words have already been introduced in the following situations:

- a. Contractions must be used in a word or part of a word joined to a numeral or to a letter by the hyphen or the dash (see §7a and §32b).
- b. The contractions for “st” and “th” must not be used in an ordinal ending attached to a mathematical expression (see §27).
- c. Contractions must not be used where contractible letter combinations occur in a mathematical sequence of letters in which each letter has a separate identity (see §33c).
- d. Contractions must not be used for the abbreviations “in.” or “in,” which usually means “inches.” The “st” contraction may only be used as the abbreviation for “street” or “saint” (see §37).
- e. The following contractions, whether capitalized, uncapitalized, or italicized, must not be used when in direct contact with a sign of grouping or with transcriber’s grouping symbols: the one-cell whole-word alphabet contractions for *but*, *can*, *do*, . . . , *as*; the whole-word lower-sign contractions for *be*, *enough*, *were*, *his*, *in*, *to*, *into*, *was*, *by*; the whole- or part-word contractions for *and*, *for*, *of*, *the*, *with*. Nor may these contractions be used when they are separated from a sign of grouping by a punctuation mark (see §50 and §57c).
- f. Contractions must not be used in a combination of letters selected to represent a sign of shape or shape modification (see §114 and §116).
- g. Contractions must not be used in a capitalized or uncapitalized word, part of a word, or abbreviation in direct contact with the symbol for the slash line. Contractions may be used if the word, part of a word, or abbreviation is separated from the slash by a sign of grouping which permits the use of the contraction, or by a punctuation mark (see §137b).
- h. Contractions must not be used in abbreviated function names. An unabbreviated function name must not be contracted in a mathematical context. In particular, the word “arc” must not be contracted when immediately preceded or followed by mathematical symbols (see §144).

§155. Additional Rules for Contractions and Short-Form Words:

- a. Contractions and short-form words must not be used in a capitalized or uncapitalized word, part of a word, or abbreviation in direct contact with any item listed below or separated from a sign of comparison by a space. This rule applies even when the word or abbreviation is separated from an item by transition to another braille line.

If an expression contains a hyphen or a dash, only the word, part of a word, or abbreviation in direct contact with one of the listed items is subject to this rule.

Contractions may be used in a word, part of a word, or abbreviation separated from any of the listed items by a punctuation mark or a sign of grouping, or in other situations which permit the use of contractions.

- i. Contractions and short-form words must not be used in a capitalized or uncapitalized word, part word, or abbreviation in direct contact with any braille indicator other than the capitalization indicator or the italic sign used in English braille.

(1) seven²

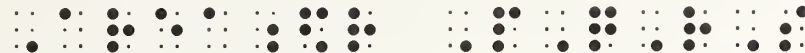
(2) seven₃



(3) 44_{four} — 35_{four} = ?_{four}



(4) Area \square_{PQRS}



(5) (ten²) + 1



(6) (ten)² + 1



(7) $\frac{\text{The length of Side AB}}{\text{The length of Side CD}}$



(8) $\overline{\text{velocity}}$



(9) heat



(10) (ten)



(11) 



(a picture of a piece of cheese is shown)

(12) Use a 5-ampere *electric current*.



(13) 1 inch-pound²



(14) 100 revolutions-min²



ii. Contractions and short-form words must not be used in a capitalized or uncapitalized word, part of a word, or abbreviation in direct contact with a numeral.

(1) 2 Arc AB cosecant $-\frac{7}{5}$



(2) $2 \sin x$



iii. Contractions and short-form words must not be used in a capitalized or uncapitalized word, part of a word, or abbreviation in direct contact with a single letter or with a sequence of letters in which each letter has a separate identity.

(1) a arc sin x



(2) $xy \sin z$



iv. Contractions and short-form words must not be used in a word, part of a word, or abbreviation in direct contact with a sign of operation, the radical sign, the general omission symbol, or any other mathematical symbol.

(1) $\text{rate} \times \text{time}$



(2) (Rate \times Time)



(3) $(\text{rate}) \times (\text{time})$



(4) 1 + seven + 1 + seven



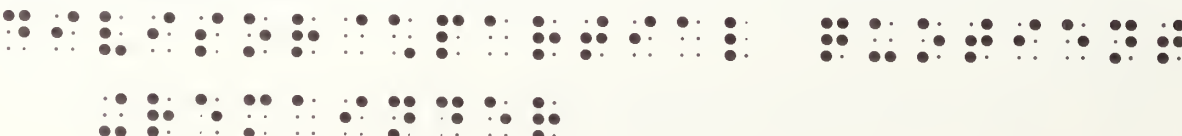
(5) nine — seven = two



(6) $(4 \times \text{seven}) + (5 \times \text{one})$



(7) divisor \times partial quotient \pm remainder



(10) $\text{rate} \times \text{time} = \text{distance}$

⠠⠗⠁⠞⠑ ⠠⠔⠊⠍⠑ ⠐⠇⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

(11) $(\text{rate}) \times (\text{time}) = (\text{distance})$

⠠⠗⠁⠞⠑ ⠠⠔⠊⠍⠑ ⠐⠇⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

(12) $\text{Length} \times \text{Width} = \text{Area}$

⠠⠇⠑⠝⠗⠞⠑ ⠠⠕⠞⠞⠑⠗ ⠐⠇⠑ ⠠⠁⠗⠑⠁

(13) $\{\text{all } x \mid \text{each } x < 8\}$

⠠⠗⠁⠞⠑ ⠠⠔⠊⠍⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

(14) What is the meaning of \cong ?

⠠⠗⠁⠞⠑ ⠠⠔⠊⠍⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

(15) $\text{Length of an arc} = \frac{n}{360} \times 2\pi r$

⠠⠇⠑⠝⠗⠞⠑ ⠠⠔⠊⠍⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

(16) It is a fundamental principle that '='s added with '='s are =.

⠠⠇⠑⠝⠗⠞⠑ ⠠⠔⠊⠍⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

(17) Use ">" and "<" in each sentence.

⠠⠇⠑⠝⠗⠞⠑ ⠠⠔⠊⠍⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

b. The contractions for *to*, *into*, and *by* must not be used before any of the items listed below. When the contraction for the word "into" cannot be used, the contraction for "in" may be used in the word "into" unless prohibited by other rules of the code.

i. The contractions for *to*, *into*, and *by* must not be used before any braille indicator except capitalization indicators or the italic sign used according to the rules of English braille.

(1) Count to 100 by *two's*.

⠠⠇⠑⠝⠗⠞⠑ ⠠⠔⠊⠍⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

(2) Ann divided the pie into 4 parts.

⠠⠇⠑⠝⠗⠞⠑ ⠠⠔⠊⠍⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

(3) Join A to B and C to D.

⠠⠇⠑⠝⠗⠞⠑ ⠠⠔⠊⠍⠑ ⠠⠔⠊⠎⠞⠁⠝⠇⠑

- (4) ab is parallel to cd .



- (5) Name the letters from α to ω .



- (6) The vector is denoted by \mathbf{i} .



- (7) Divide $\frac{1}{2}$ by $\frac{1}{4}$.



- (8) Add -7 to -4 .



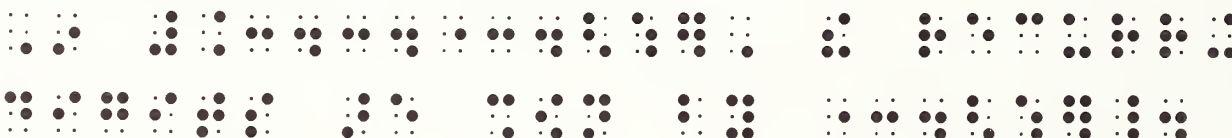
- (9) $\angle A$ is equal to $\angle B$.



- (10) Is \overline{AB} equal to \overline{cd} ?



- (11) In $.3434\overline{34}$, the recurring digits are shown by $\overline{34}$.



ii. The contractions for *to*, *into*, and *by* must not be used before a Roman numeral, a single letter, or a sequence of letters in which each letter has a separate identity, whether or not the Roman numeral, the single letter, or the sequence of letters requires the English letter indicator.

- (1) The imaginary part is denoted by $i = (0, 1)$.



- (2) Is $A : B$ equivalent to $C : D$?



- (3) Draw a line to pq .



- (4) AB is parallel to CD.



- (5) Is $a \cdot b$ equal to $b \cdot a$?



- (6) Use numerals showing that 10×10 is equal to $ab \times ba$.



- (7) Explain the change from \bar{x} to \bar{y} .



- (8) Read Chapters II to VI and IX to X.



- (9) Solve examples i to iv.



- (10) 17 is equivalent to VII + X.



- (11) 17 is equivalent to vii + x.

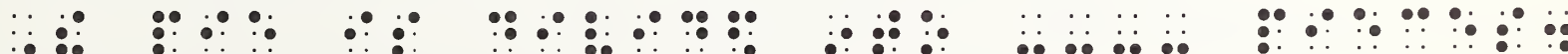


iii. The contractions for *to*, *into*, and *by* must not be used before a dash, the ellipsis, or the general omission symbol.

- (1) 50 divided by ____ equals 25.



- (2) The pie is divided into ____ pieces.



- (3) 31 added to ... equals 42.



- (4) 6×4 divided by ... = 8.



- (5) 63 divided by ? = 7.



- (9) The empty set is denoted by \emptyset or by $\{ \}$.



- (10) $1/x$ tends to ∞ as $x \rightarrow 0$.



- (11) The indefinite integral is to be denoted by $\int f(x) dx$.



- (12) The numeral 3 is represented by ///.

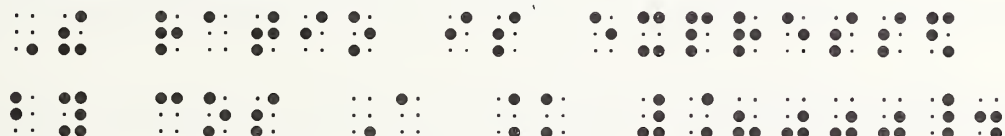


vi. The contractions for *to*, *into*, and *by* must not be used before any abbreviated function name or before an unabbreviated function name in mathematical context.

- (1) y is proportional to $\log x$.



- (2) The ratio is expressed by $\cos A = .8000$.



- (3) The ratio is expressed by $\cosine A = .8000$.



vii. The contractions for *to*, *into*, and *by* must not be used before an abbreviation.

- (1) Turn to p. 46 to learn to convert 50 kg. into lbs.



- (2) Convert mm to m.



- (3) Convert 730 days into yrs. to obtain an answer.

730 days into yrs. to obtain an answer.

viii. The contractions for *to*, *into*, and *by* must not be used before a word, part of a word, or abbreviation in a context in which contractions are not permitted by any of the other rules of the code.

- (1) people who go by car + people who go by train

people who go by car + people who go by train

- (2) the people who travel by car/the people who travel by train

the people who travel by car/the people who travel by train

- (3) The area is divided into in², and not into ft².

The area is divided into in², and not into ft².

- (4) Change > to < to make a true statement.

Change > to < to make a true statement.

- (5) Replace ? by = or ≠ to make a true sentence.

Replace ? by = or ≠ to make a true sentence.

- (6) Compare Seven₂ to Seven₃.

Compare Seven₂ to Seven₃.

- (7) Multiply ten⁵ by ten².

Multiply ten⁵ by ten².

- (8) The distance is equal to time × rate.

The distance is equal to time × rate.

c. Contractions must not be used when they can be mistaken for mathematical expressions or symbols.

(1) Use the \int to find the volume.

(2) Can $C = 100$?

(3) $a = b$, but $b \neq c$.

(4) We see that $x = y$.

(5) We know $<$ means “is less than”.

FORMAT (CONTINUED)

§156. Special Format Requirements for Linked Expressions: A linked expression must contain at least one sign of comparison. The part preceding the first sign of comparison is called the anchor. Each remaining part, beginning with a sign of comparison and ending before the next sign of comparison, is called a link.

A linked expression is subject to special braille format requirements if it appears in print in the following way:

a. The expression is displayed and not embedded in the text.

b. Its signs of comparison are vertically aligned, except possibly for the last few, which may occur on the last print line of the expression.

c. No sign of comparison is preceded by any expression on its left, excepting the anchor on the first line and any links on the last line.

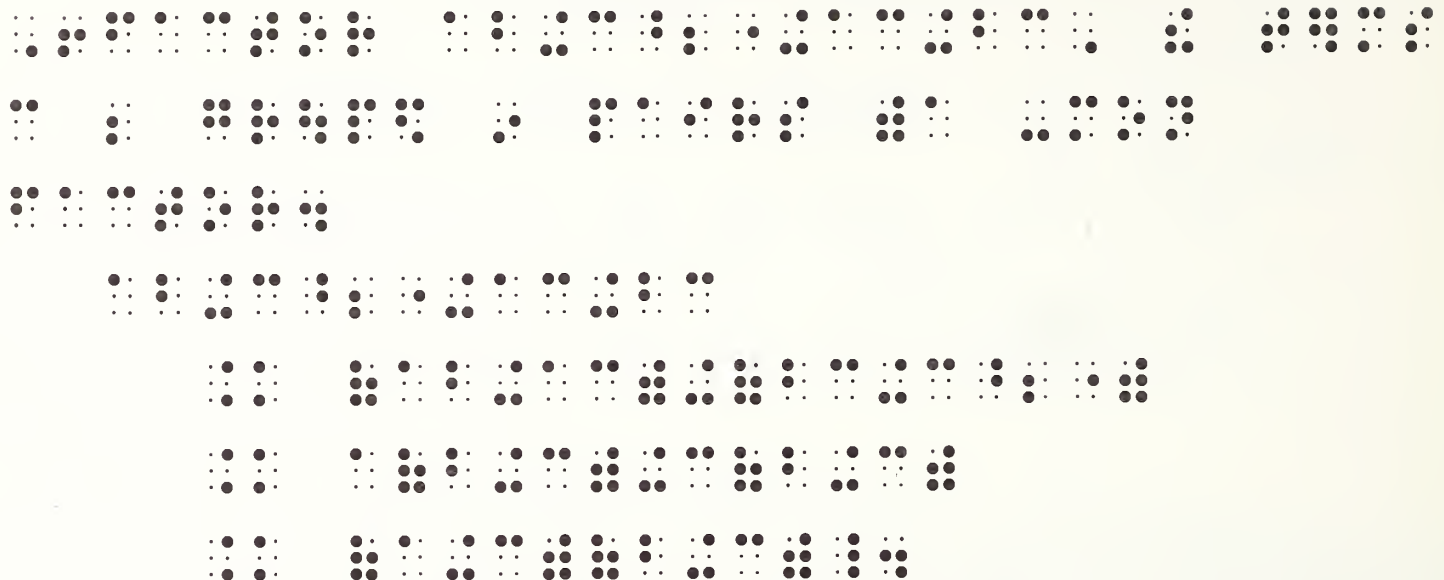
$$\begin{aligned} 33\frac{1}{3}\% &= \frac{33\frac{1}{3}}{100} \\ &= \frac{3 \times 33\frac{1}{3}}{3 \times 100} \\ &= \frac{100}{300} = \frac{1}{3} \end{aligned}$$

§157. Braille Format for Special Linked Expressions:

a. Under the special format requirements, when a linked expression occurs in unitemized explanatory portions of the text, the anchor must begin in cell 3, and its runovers must begin in cell 7. Each link must begin in cell 5, and its runovers must begin in cell 7.

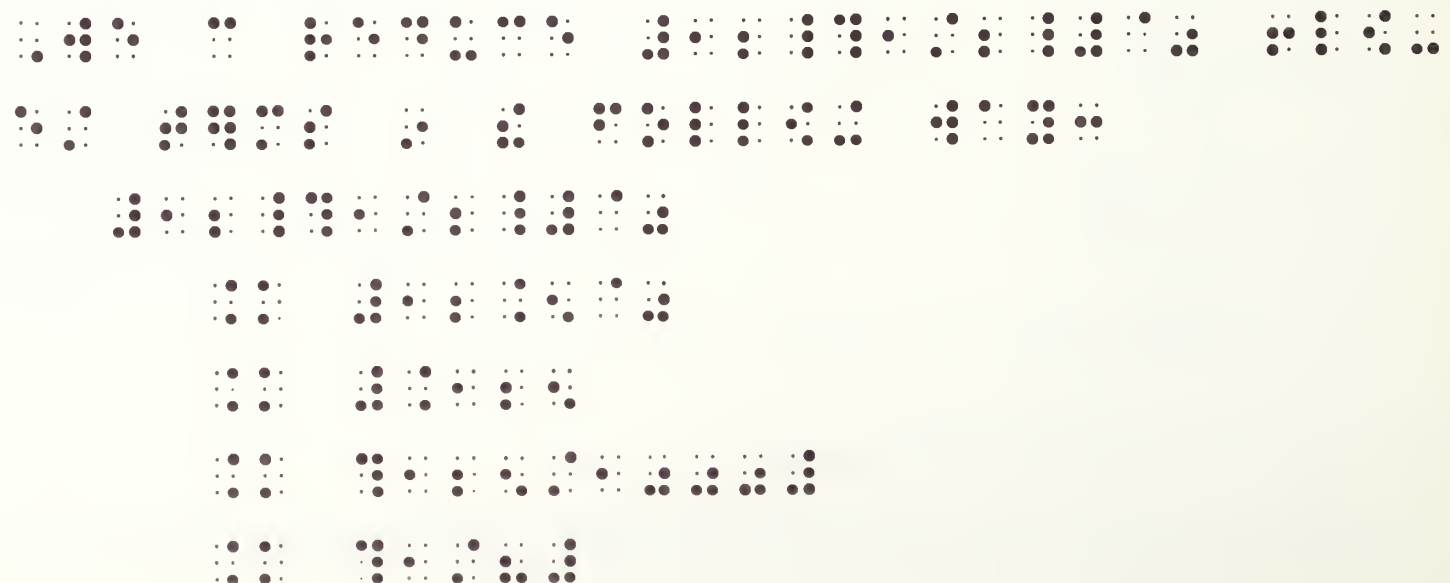
- (1) To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.

$$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c). \end{aligned}$$



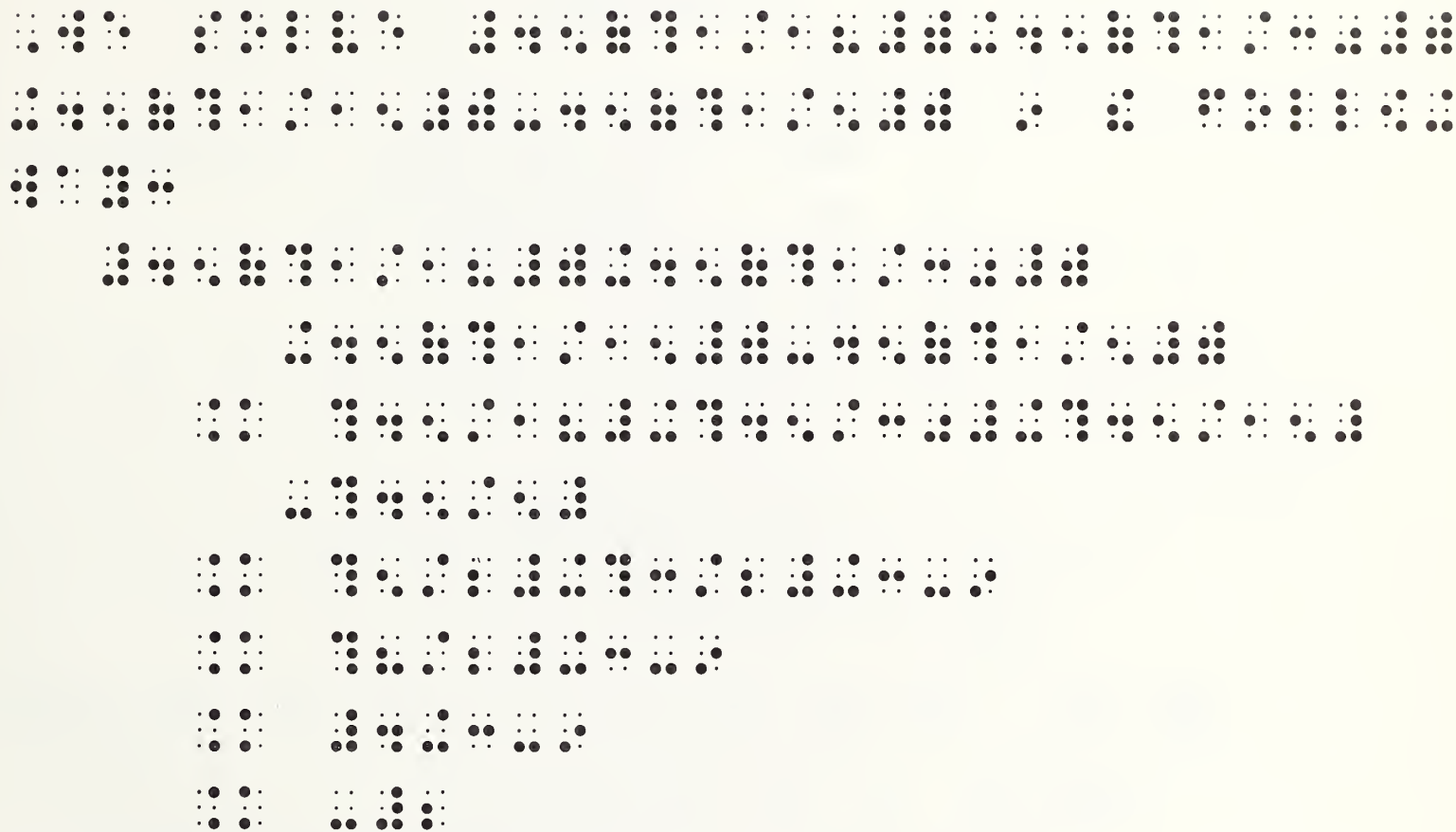
- (2) We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:

$$\begin{aligned} 12\frac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \frac{125}{1000} = \frac{1}{8} \end{aligned}$$



(3) We solve $45\left(\frac{1}{18}\right) + 45\left(\frac{1}{30}\right) + 45\left(\frac{1}{15}\right) - 45\left(\frac{1}{5}\right)$ in the following way:

$$\begin{aligned} &45\left(\frac{1}{18}\right) + 45\left(\frac{1}{30}\right) + 45\left(\frac{1}{15}\right) - 45\left(\frac{1}{5}\right) \\ &= \frac{45}{18} + \frac{45}{30} + \frac{45}{15} - \frac{45}{5} \\ &= \frac{5}{2} + \frac{3}{2} + 3 - 9 \\ &= \frac{8}{2} + 3 - 9 = 4 + 3 - 9 = -2 \end{aligned}$$



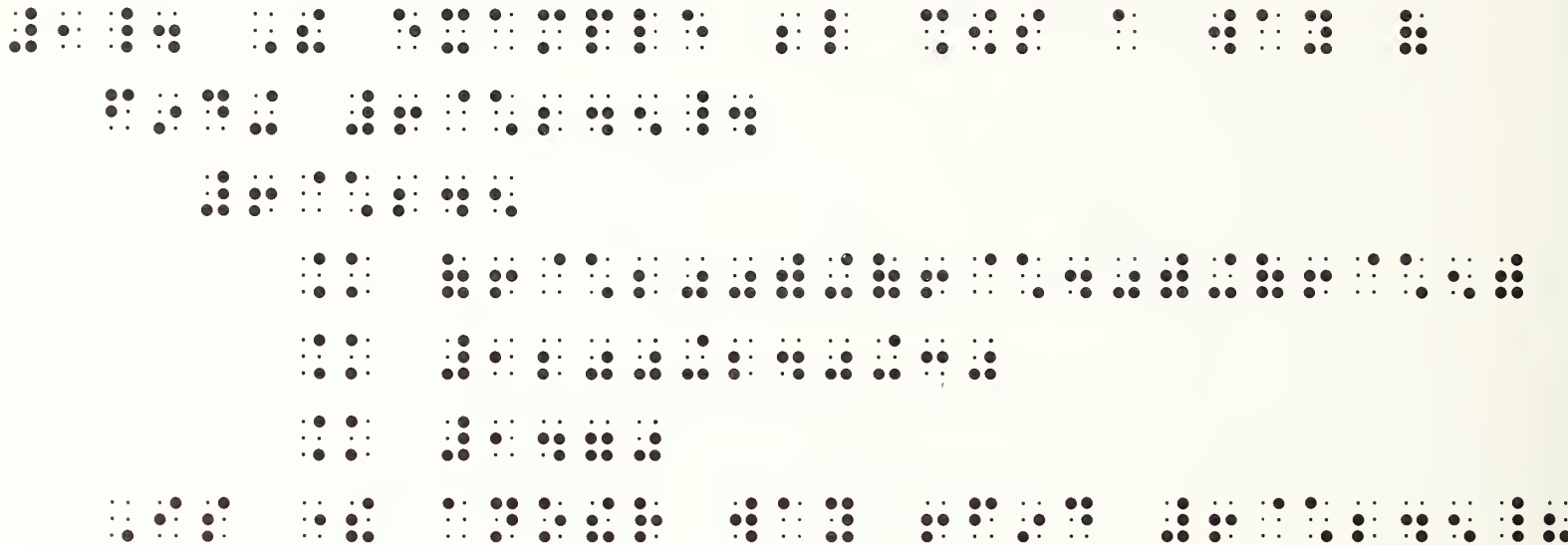
(In print, the last three links appear on the same line).

b. In an itemized text containing no subdivisions, the anchor must begin in cell 5, and its runovers must begin in cell 9. Each link must begin in cell 7, and its runovers must begin in cell 9.

(1) 1. The example below shows a way of finding 6×245 .

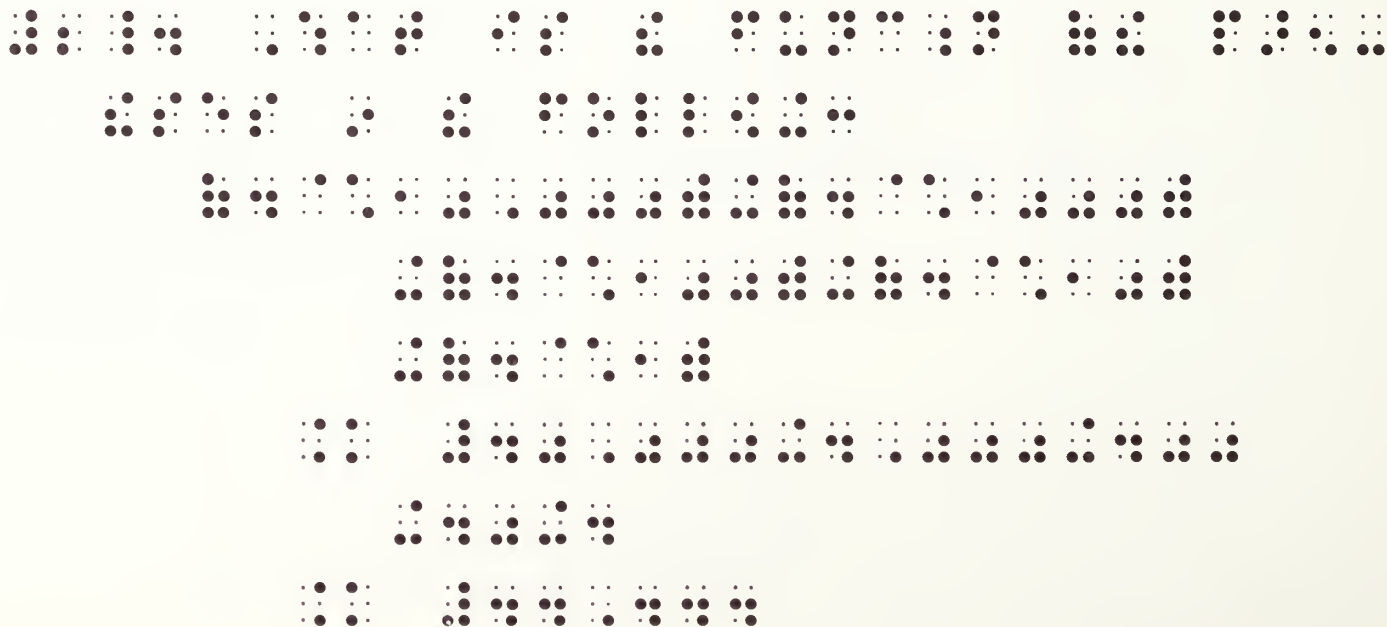
$$\begin{aligned} 6 \times 245 &= (6 \times 200) + (6 \times 40) + (6 \times 5) \\ &= 1200 + 240 + 30 \\ &= 1470 \end{aligned}$$

Is there another way to find 6×245 ?



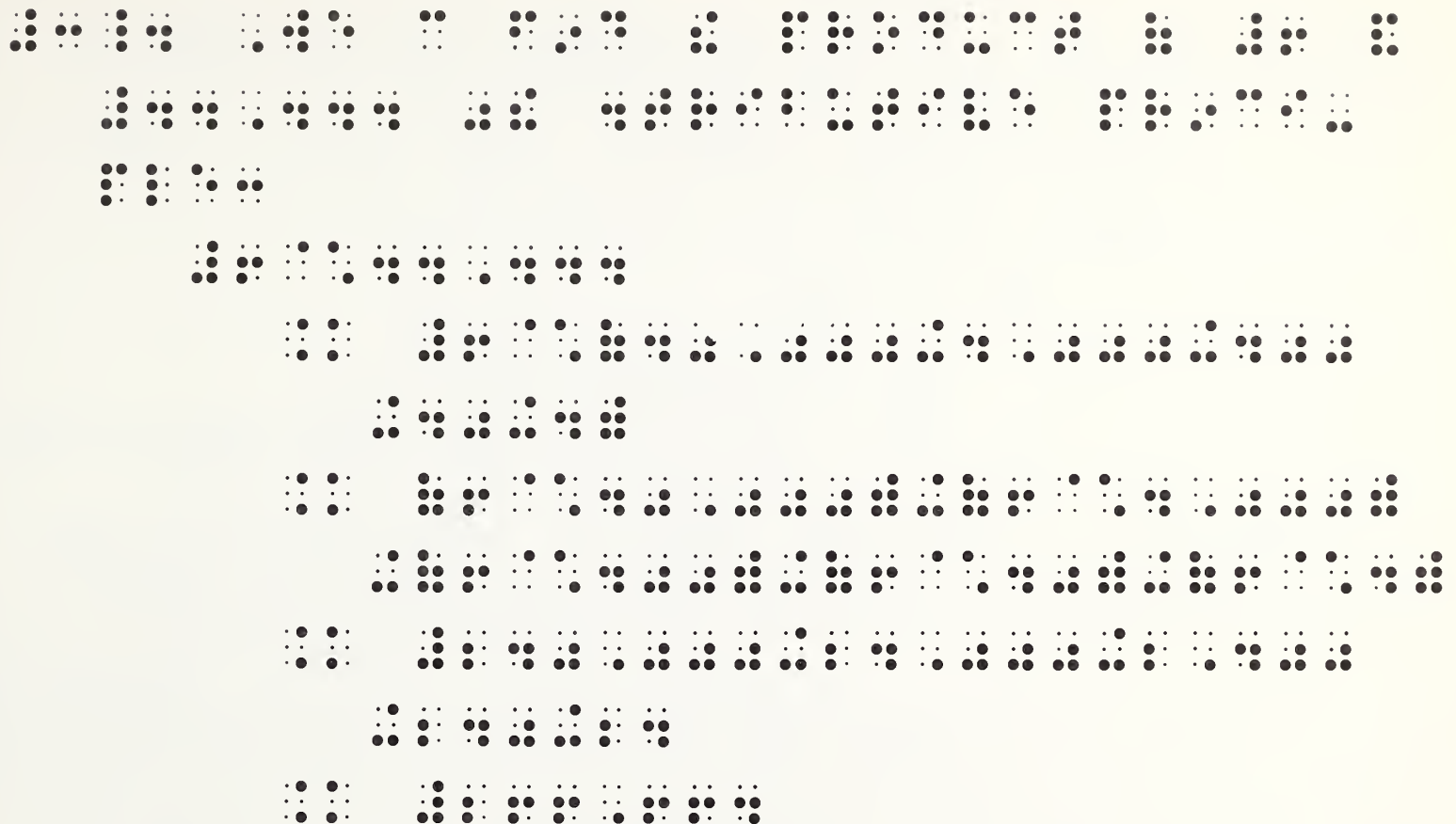
(2) 2. What is the function of the parentheses in the following:

$$\begin{aligned} (4 \times 10,000) + (4 \times 1000) + (4 \times 100) + (4 \times 10) + (4 \times 1) \\ = 40,000 + 4,000 + 400 + 40 + 4 \\ = 44,444 \end{aligned}$$



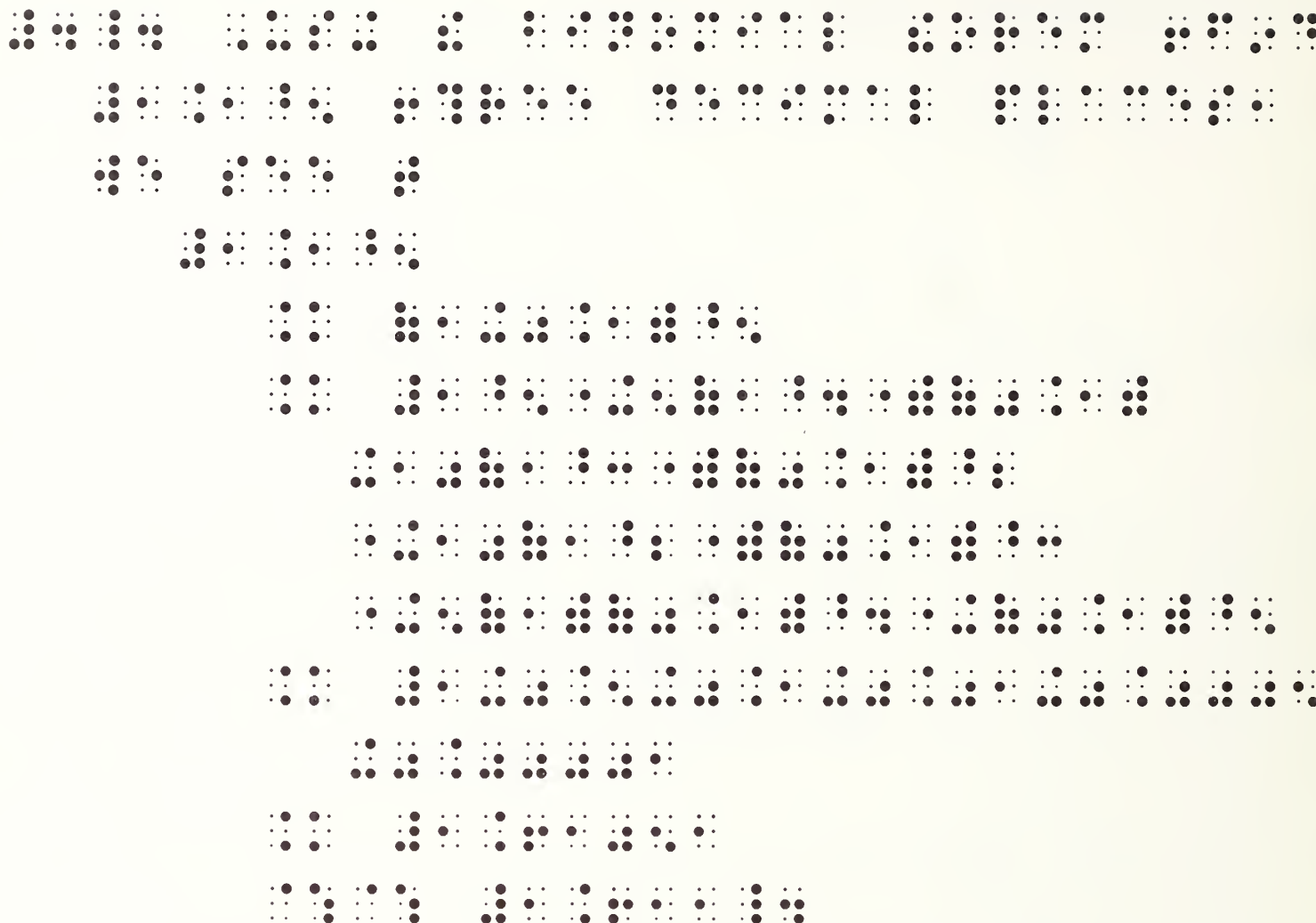
(3) 3. We can find the product of 6 and 44,444 by the distributive principle:

$$\begin{aligned}
 6 \times 44,444 &= 6 \times (40,000 + 4,000 + 400 + 40 + 4) \\
 &= (6 \times 40,000) + (6 \times 4,000) + (6 \times 400) + (6 \times 40) + (6 \times 4) \\
 &= 240,000 + 24,000 + 2,400 + 240 + 24 \\
 &= 266,664
 \end{aligned}$$



(4) 4. Using the binomial theorem to find 1.1^5 to three decimal places, we see that

$$\begin{aligned}
 1.1^5 &= (1 + 0.1)^5 \\
 &= 1^5 + 5(1^4)(0.1) + 10(1^3)(0.1)^2 + \\
 &\quad 10(1^2)(0.1)^3 + 5(1)(0.1)^4 + (0.1)^5 \\
 &= 1 + 0.5 + 0.1 + 0.01 + 0.0005 + 0.00001 \\
 &= 1.61051 \approx 1.611.
 \end{aligned}$$



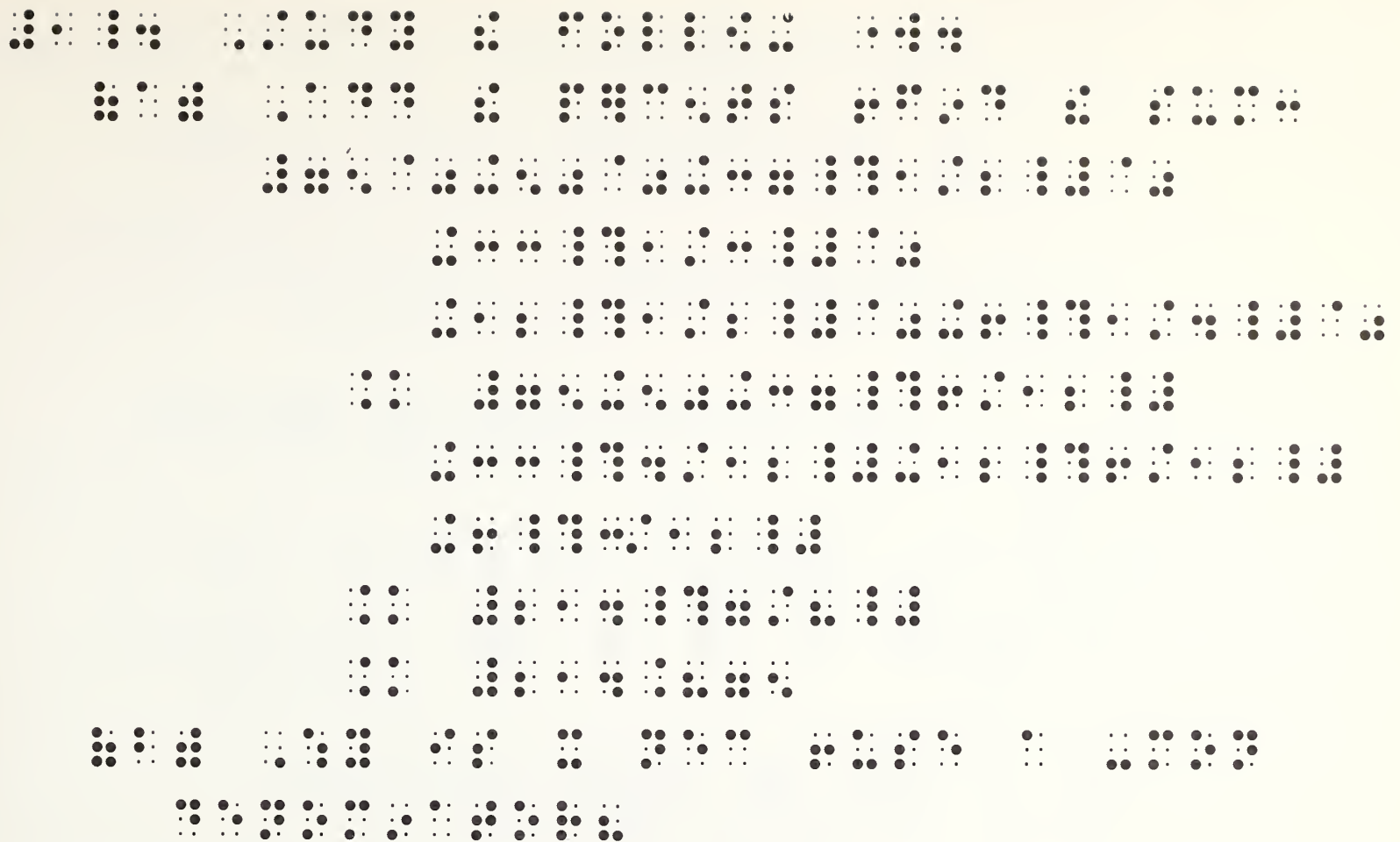
c. In an itemized text containing subdivisions, the anchor must begin in cell 7, and its runovers must begin in cell 11. Each link must begin in cell 9, and its runovers must begin in cell 11.

(1) 1. Study the following work.

(a) Add the percents to find the sum:

$$\begin{aligned}
 75\% + 50\% + 37\frac{1}{2}\% + 33\frac{1}{3}\% + 12\frac{1}{2}\% + 6\frac{1}{4}\% \\
 &= 75 + 50 + 37\frac{6}{12} + 33\frac{4}{12} + 12\frac{6}{12} + 6\frac{3}{12} \\
 &= 214\frac{7}{8} = 214.875
 \end{aligned}$$

(b) Why is it necessary to use a common denominator?

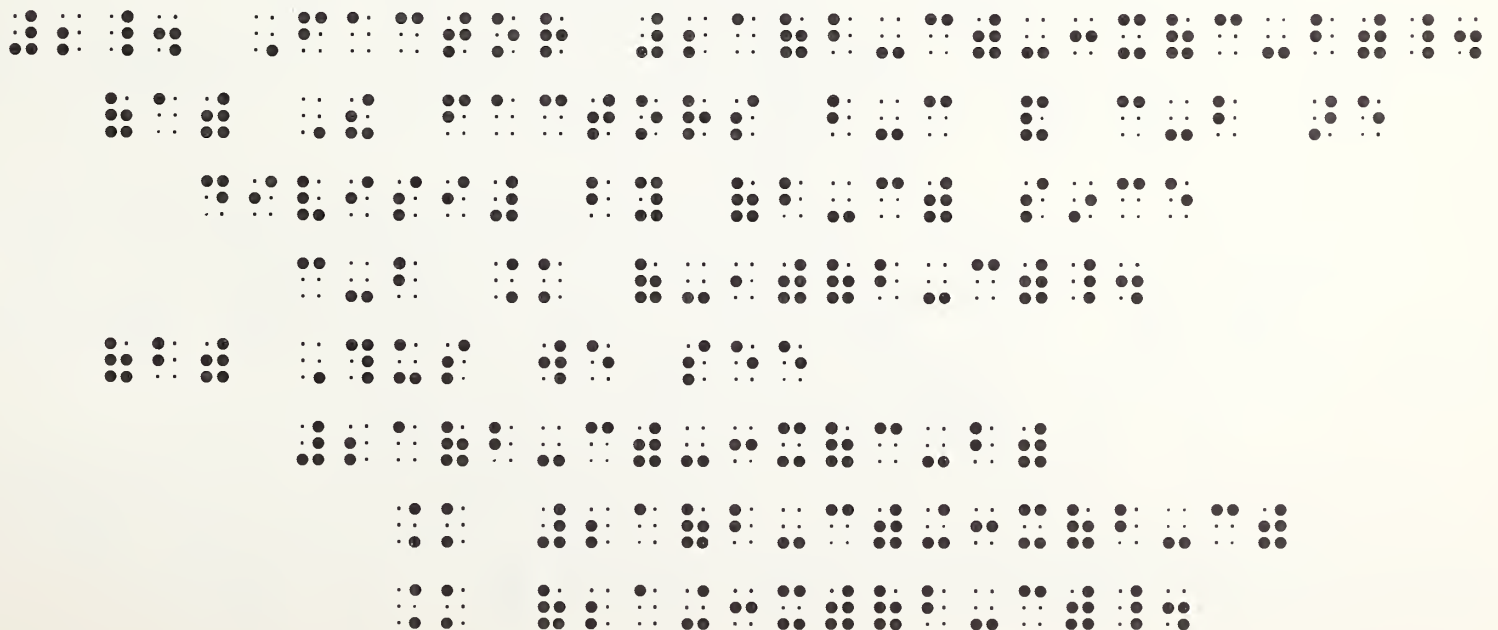


(2) 2. Factor $2a(b - c) - 3x(c - b)$.

(a) The factors $b - c$ and $c - b$ are divisible by $(b - c)$ since
 $c - b = (-1)(b - c)$.

(b) Thus we see

$$\begin{aligned} 2a(b - c) - 3x(c - b) &= 2a(b - c) + 3x(b - c) \\ &= (2a + 3x)(b - c). \end{aligned}$$



(3) 3. Study the following for given $\triangle ABC$.

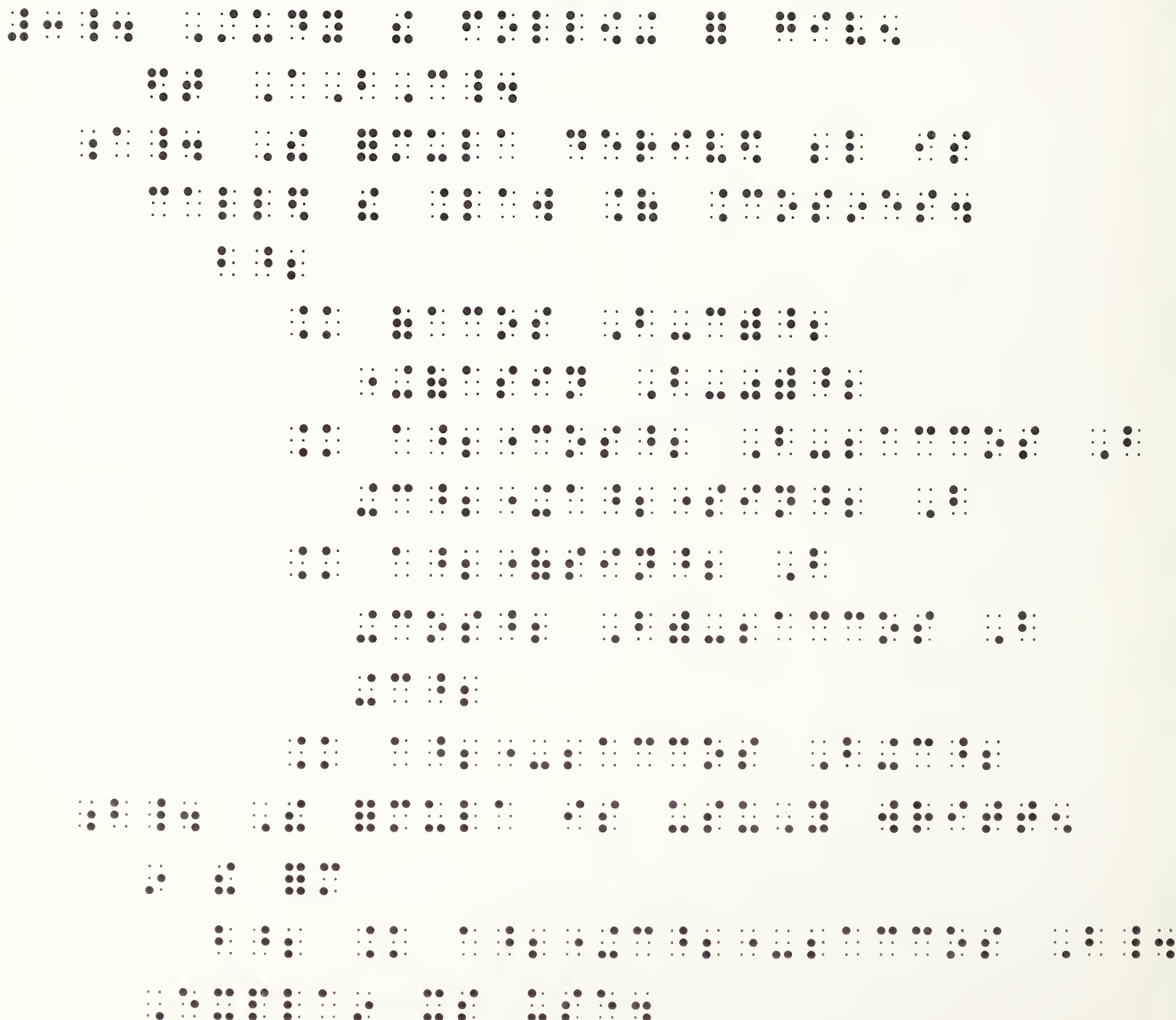
a. The formula derived below is called the *law of cosines*.

$$\begin{aligned} b^2 &= (a \cos B - c)^2 + (a \sin B - 0)^2 \\ &= a^2 \cos^2 B - 2ac \cos B + c^2 + a^2 \sin^2 B \\ &= a^2 (\sin^2 B + \cos^2 B) - 2ac \cos B + c^2 \\ &= a^2 - 2ac \cos B + c^2 \end{aligned}$$

b. The formula is usually written in the form

$$b^2 = a^2 + c^2 - 2ac \cos B.$$

Explain its use.



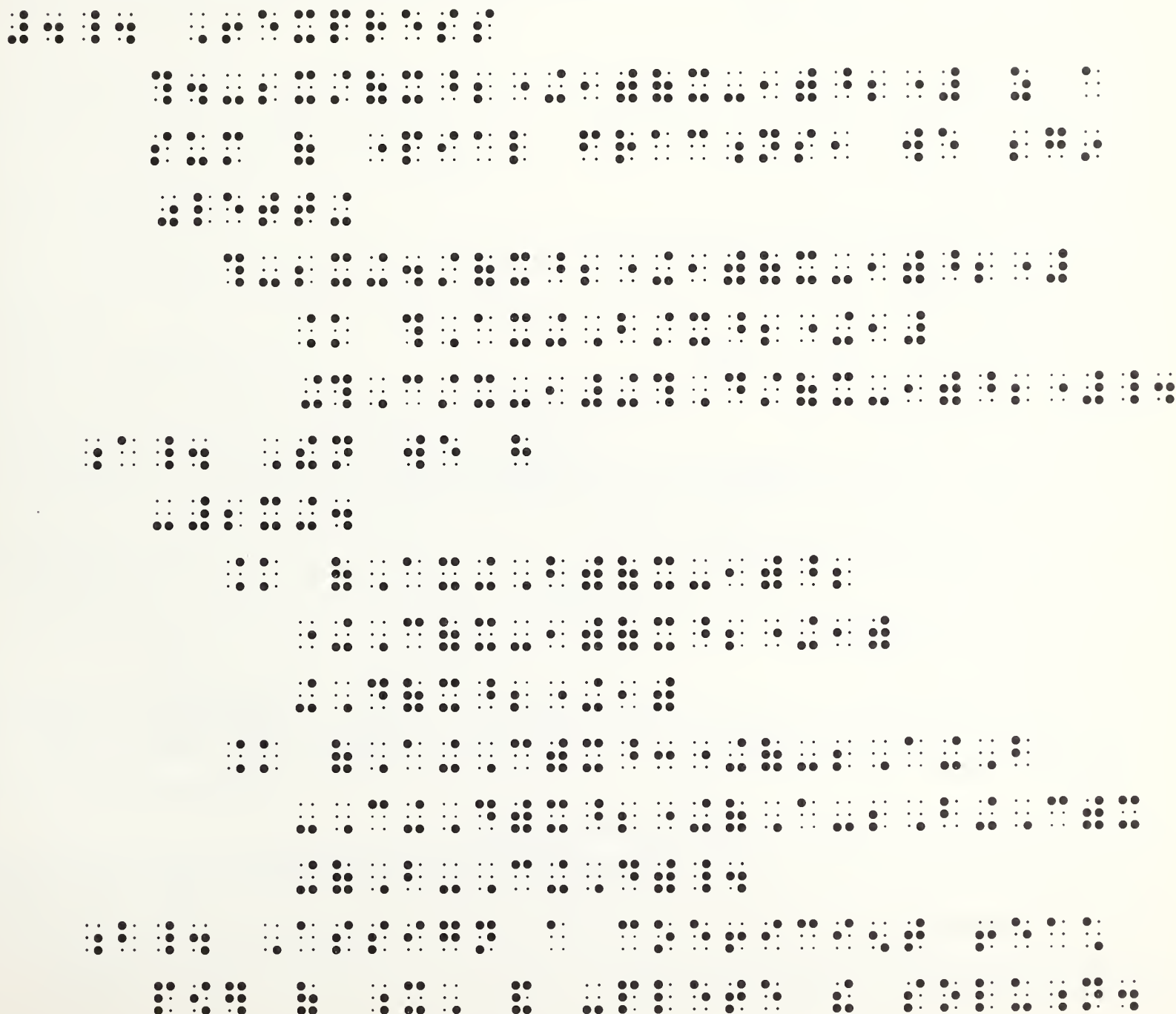
(4) 4. To express $\frac{4 - 2x}{(x^2 + 1)(x - 1)^2}$ as a sum of partial fractions, we begin by letting

$$\frac{-2x + 4}{(x^2 + 1)(x - 1)^2} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x - 1} + \frac{D}{(x - 1)^2}.$$

a. Then we have

$$\begin{aligned} -2x + 4 &= (Ax + B)(x - 1)^2 + C(x - 1)(x^2 + 1) + D(x^2 + 1) \\ &= (A + C)x^3 + (-2A + B - C + D)x^2 + (A - 2B + C)x + (B - C + D). \end{aligned}$$

b. Assign a coefficient to each power of x , and complete the solution.



HOMEWORK

Prepare the following homework for submission to your teacher. Proofread carefully.

EXERCISE 14

1. If $\text{eight}^2 \times \text{eight}^3 = \text{eight}^5$, what is the product of $\text{nine}^{18} \times \text{nine}^7$?
2. What is the meaning of the phrase “inches-hour²”?
3.

twenty

 +

four

 =

○

4. Is the following true: In a right triangle
 $(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$?
5. Solve:
 - a. $1 \times \text{seven}^3 + 181$
 - b. $(2 \times \text{seven} \times \text{seven}) + (4 \times \text{seven}) + (6 \times \text{one})$
 - c. $30_{\text{seven}} + 1_{\text{seven}}$
 - d. $(64)_{\text{eight}} + (42)_{\text{eight}}$
6. Find the frequency of the wave whose equation is given.
 $v = 5 \sin 120\pi t$
7. Prove the identity.
 $\tan x \sin x = \sec x - \cos x$
8. $\text{length} \times \text{width} = \frac{\text{volume}}{\text{height}}$
9. If 2 is the multiplier, 3 is the multiplicand, and 6 is the product, their relationship may be stated as follows:
 $\text{multiplier} \times \text{multiplicand} = \text{product}$.
Do you see that the relationship in a division example is:
 $\text{product} \div \text{multiplier} = \text{multiplicand}$?
10. The volume of a right circular cone is denoted by the formula
$$\text{Volume} = \frac{(\text{area})(\text{height})}{3}.$$
11. You can find the rate of climb of an airplane by using the formula $r = \frac{33,000p}{w}$. In this formula, if r = the rate of climb in feet per second and p = the power of the engine, will w = the weight of the airplane in pounds? (33,000 is exact.) Find r if $p = 1500$ and $w = 20,000$ (to two significant digits).
12. The 2 boys can count to 1,000 and the 3 girls can count to 10,000.
13. Multiply 6 by 2, by 3, by 4, and so on until you find a number that is a multiple of 3 and 5.
14. Use the number line to add $+4$ to $+7$.
15. Instead of dividing 6 by $\frac{1}{2}$, can you multiply 6 by $\frac{2}{1}$?

16. If 1 kg. is about 2.2 lb., then 1 lb. is approximately equal to .454 kg. Then a pound is equivalent to how many grams?
17. Is line AB equal to CD? Is line ab equal to cd?
18. Use numerals and the symbols $>$, $<$ to write two true statements about the cardinal numbers in the pair of sets below:
 $R = \{\text{Roger, Robert, Richard, Rufus}\}$
 $S = \{\text{Sally, Sandra, Sylvia, Sarah}\}$
19. a. Add -119 to -67 .
b. Add $-.38$ to $-.42$.
20. What is the value assigned to π in section 15-3 of the text?
21. Find the answer in sections VI to X.
22. Find the quotients and remainders.
a. 405 into 815,245 is equal to _____ with remainder _____.
b. 588 into 2,755,000 is equal to _____ with remainder _____.
c. 118,422 divided by 6025 is equal to ... with remainder
23. In §1 to §5 we studied fractions.
24. $(7 + 3)$ to $(7 + 5)$ are names for certain numerals. Find the three names and then multiply each by (-7) .
25. Divide $\sqrt{4}$ by $\sqrt{9}$.
26. The function of x is denoted by $\cos x$.
27. Turn 657 mins. into hrs. and then turn these hrs. into parts of days.

To express $(-1 + i)^{-4}$ in the form $a + bi$, we first express $-1 + i$ in polar form:

$$-1 + i = \sqrt{2} (\cos 135^\circ + i \sin 135^\circ).$$

We then apply the extended DeMoivre's theorem to obtain

$$\begin{aligned} (-1 + i)^{-4} &= (\sqrt{2})^{-4} [\cos(-4 \cdot 135^\circ) + i \sin(-4 \cdot 135^\circ)] \\ &= \frac{1}{4} [\cos(-540^\circ) + i \sin(-540^\circ)] \\ &= \frac{1}{4} (\cos 180^\circ + i \sin 180^\circ) = -\frac{1}{4} + 0i. \end{aligned}$$

$$\therefore (-1 + i)^{-4} = -\frac{1}{4} + 0i \text{ is the answer.}$$

28. The distributive axiom enables you to simplify products of radical expressions.

$$\begin{aligned} &3\sqrt{2} (5\sqrt{3} + 2\sqrt{5}) \\ &= (3\sqrt{2}) (5\sqrt{3}) + (3\sqrt{2}) (2\sqrt{5}) \\ &= 15\sqrt{6} + 6\sqrt{10} \end{aligned}$$

29. Find the distance between A(−1, 3) and B(4, −6).

a. The distance between A and B is

$$\begin{aligned}d(A, B) &= \sqrt{[4 - (-1)]^2 + [(-6) - 3]^2} \\&= \sqrt{(5)^2 + (-9)^2} \\&= \sqrt{106}.\end{aligned}$$

b. Similarly, the distance between B and A is shown to be

$$\begin{aligned}d(B, A) &= \sqrt{[(-1) - 4]^2 + [3 - (-6)]^2} \\&= \sqrt{(-5)^2 + (9)^2} \\&= \sqrt{106}.\end{aligned}$$

LESSON 15

INTRODUCTION TO SPATIAL ARRANGEMENTS

§158. Spatial Arrangements: Spatial arrangement is required with material arrayed on more than one line in print, as in addition, subtraction, multiplication, division, and determinants and matrices.

§159. Blank Lines With Spatial Arrangements: One blank line must be left above and below a spatial arrangement, even when it directly precedes or follows the line indicating a new ink-print page. If a running head is used, a blank line must be left between it and the spatial arrangement. However, no blank line is required if the spatial arrangement begins at the top or ends at the bottom of the braille page.

§160. Nonuse of Numeric Indicator: The numeric indicator must not be used in aligned arrangements of columns for addition, subtraction, multiplication, division, or other material aligned for computation.

§161. Spatial Arrangements With Addition and Subtraction:

a. The separation line is composed of a line of dots 2-5. The line must extend one cell to the left and to the right of the longest entry appearing above or below it.

(1)
$$\begin{array}{r} 24 \\ 35 \\ \hline 59 \end{array}$$

$$\begin{array}{r} \begin{smallmatrix} \cdot\cdot & \cdot\cdot \\ \cdot\cdot & \cdot\cdot \end{smallmatrix} \\ \begin{smallmatrix} \cdot\cdot & \cdot\cdot \\ \cdot\cdot & \cdot\cdot \end{smallmatrix} \\ \hline \begin{smallmatrix} \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \end{smallmatrix} \\ \begin{smallmatrix} \cdot\cdot & \cdot\cdot \\ \cdot\cdot & \cdot\cdot \end{smallmatrix} \end{array}$$

(2)
$$\begin{array}{r} 900 \\ 125 \\ \hline 1025 \end{array}$$

$$\begin{array}{r} \begin{smallmatrix} \cdot\cdot & \cdot\cdot & \cdot\cdot \\ \cdot\cdot & \cdot\cdot & \cdot\cdot \end{smallmatrix} \\ \begin{smallmatrix} \cdot\cdot & \cdot\cdot & \cdot\cdot \\ \cdot\cdot & \cdot\cdot & \cdot\cdot \end{smallmatrix} \\ \hline \begin{smallmatrix} \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \end{smallmatrix} \\ \begin{smallmatrix} \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \end{smallmatrix} \end{array}$$

b. In spatial arrangements for addition and subtraction, the corresponding digits, commas, decimal points, fractions, and interior signs of operation or comparison must be vertically aligned one below the other unless they have been intentionally misaligned as an exercise for the student.

(1)
$$\begin{array}{r} .36 \\ 7.02 \\ 3.04 \\ \hline \end{array}$$

(2)
$$\begin{array}{r} 3,854 \\ 602 \\ 5,918 \\ \hline \end{array}$$

(3)
$$\begin{array}{r} 4x+20y+6z \\ 18x \quad \quad -9z \\ 5y+4z \\ \hline \end{array}$$

(4) Arrange the digits for proper place value.

25.92

10.4

3.796

c. A plus, minus, or dollar sign must be placed in the same position as in ink-print. The dollar sign or sign of operation must be placed to the left, one cell beyond the largest numeric entry appearing *above* the separation line. If there is an answer, part of it may be shown beneath the plus, minus, or dollar sign.

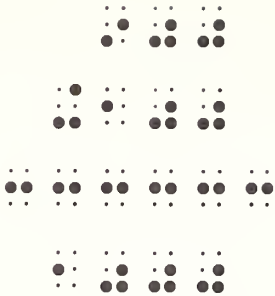
(1) 4391

+ 81

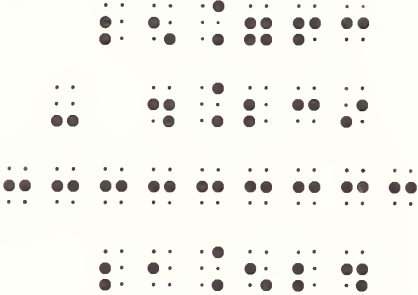
(2) 718

—437

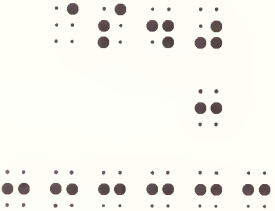
(3) 900
 +100
 ——
 1000



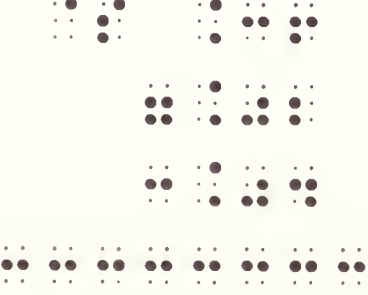
(4) 25.763
 — 4.239
 ——
 21.524



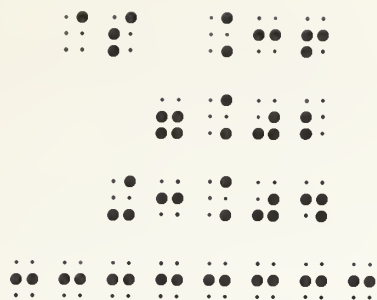
(5) \$40
 3
 ——



(6) \$.36
 7.02
 3.04
 ——

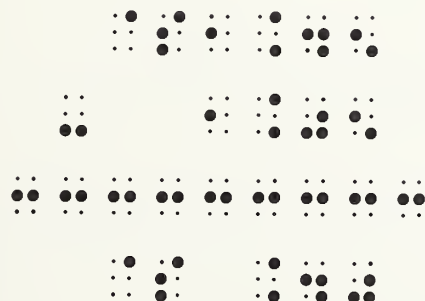


(7) $\begin{array}{r} \$.36 \\ 7.02 \\ + 3.04 \\ \hline \end{array}$



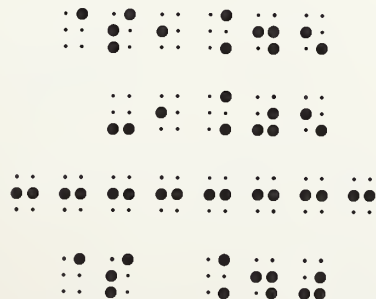
(the plus sign appears beneath the dollar sign in print)

(8) $\begin{array}{r} \$1.45 \\ - 1.05 \\ \hline \$.40 \end{array}$



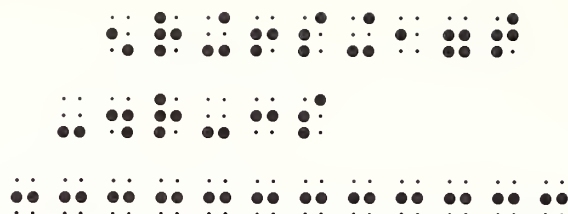
(the minus sign appears to the left of the dollar sign in print)

(9) $\begin{array}{r} \$1.45 \\ -1.05 \\ \hline \$.40 \end{array}$



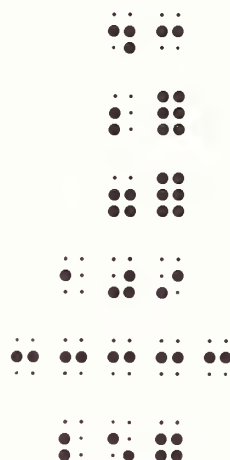
(the minus sign appears beneath the dollar sign in print)

$$(10) \quad \begin{array}{r} 5r+6s+17t \\ -4r-3s \\ \hline \end{array}$$

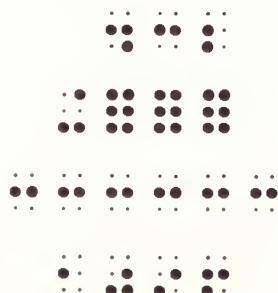


d. When spatial arrangements contain omissions, only the general omission symbol may be used. The long dash and ellipsis must never be used. The same number of omission symbols as in ink-print should be used.

$$(1) \quad \begin{array}{r} 43 \\ 2? \\ 7? \\ 109 \\ \hline 257 \end{array}$$



$$(2) \quad \begin{array}{r} 432 \\ +??? \\ \hline 1096 \end{array}$$



(3) \$7.18

$$\begin{array}{r} \text{---} \dots \\ \$5.20 \end{array}$$

$$\begin{array}{r} \dots \\ \$5.20 \end{array}$$

(the ellipsis is shown in print)

(4) $17x - 8y - z$

$$\begin{array}{r} - 2x + 17y + 6z \\ 15x + 9y + \text{---} \end{array}$$

$$\begin{array}{r} \dots \\ 15x + 9y + \text{---} \end{array}$$

(the long dash is shown in print)

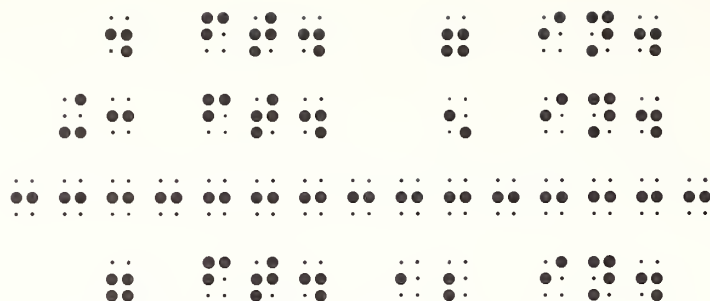
e. When abbreviations occur in spatially arranged addition and subtraction, they are vertically aligned and transcribed according to the rules for abbreviations.

(1) 4 ft 7 in

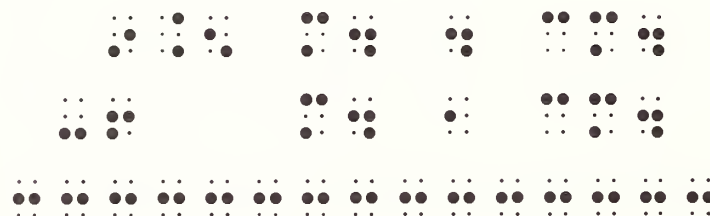
$$\begin{array}{r} + 3 \text{ ft } 5 \text{ in} \\ \text{---} \end{array}$$

$$\begin{array}{r} \dots \\ + 3 \text{ ft } 5 \text{ in} \\ \text{---} \end{array}$$

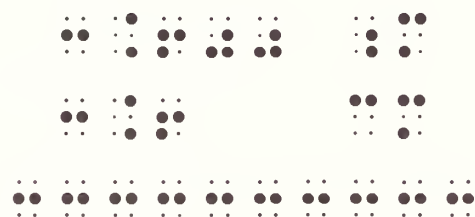
$$\begin{array}{r}
 (2) \quad 4 \text{ ft. } 7 \text{ in.} \\
 + 3 \text{ ft. } 5 \text{ in.} \\
 \hline
 7 \text{ ft. } 12 \text{ in.}
 \end{array}$$



$$\begin{array}{r}
 (3) \quad 9.5 \text{ m. } 4 \text{ cm.} \\
 - 6 \text{ m. } 1 \text{ cm.} \\
 \hline
 \end{array}$$

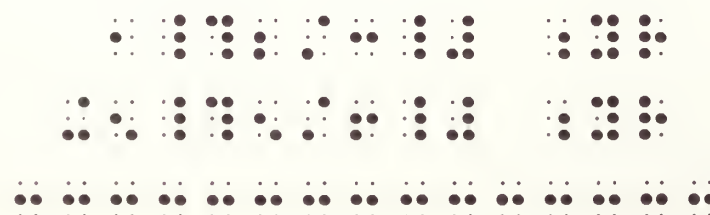


$$\begin{array}{r}
 (4) \quad 3.600 \text{ m} \\
 3.6 \text{ cm} \\
 \hline
 \end{array}$$

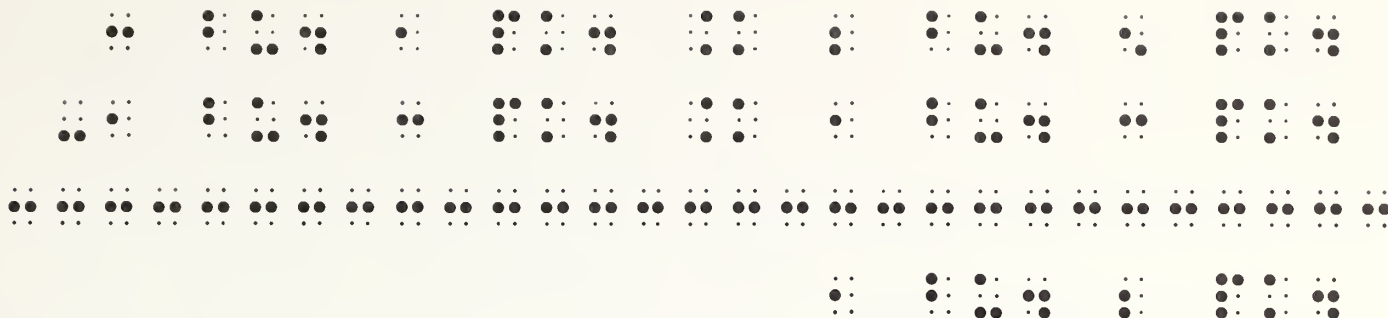


$$(5) \quad 1\frac{2}{3} \text{ yr}$$

$$+ 5\frac{5}{6} \text{ yr}$$

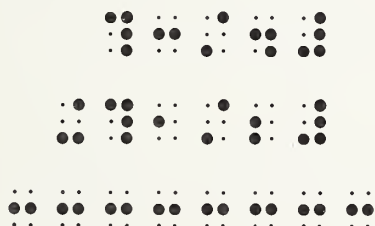


(6)
$$\begin{array}{r} 3 \text{ bu. } 1 \text{ pk.} = 2 \text{ bu. } 5 \text{ pk.} \\ - 1 \text{ bu. } 3 \text{ pk.} = 1 \text{ bu. } 3 \text{ pk.} \\ \hline 1 \text{ bu. } 2 \text{ pk.} \end{array}$$

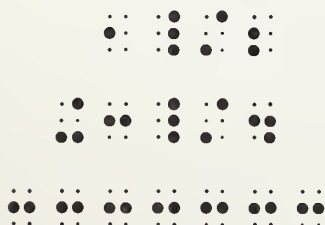


f. In a spatial addition or subtraction arrangement containing fractions, the fraction lines must be vertically aligned. The numerator and denominator must be written unspaced from the fraction line. The corresponding parts of fraction indicators must be vertically aligned. The whole number parts of mixed numbers must also be vertically aligned.

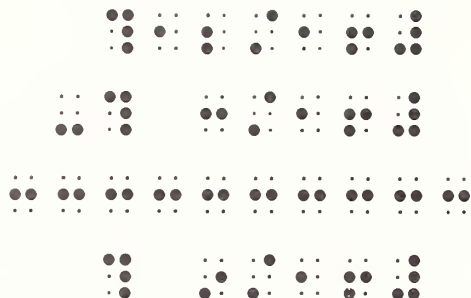
(1)
$$\begin{array}{r} \frac{3}{4} \\ + \frac{1}{2} \\ \hline \end{array}$$



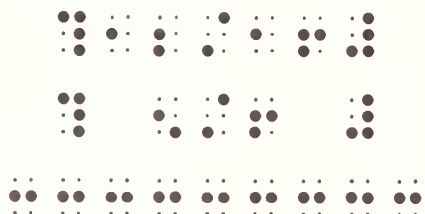
(2)
$$\begin{array}{r} \frac{1}{2} \\ + \frac{3}{4} \\ \hline \end{array}$$



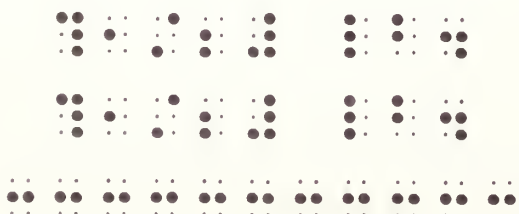
$$\begin{array}{r}
 (3) \quad \frac{12}{16} \\
 - \quad \frac{3}{16} \\
 \hline
 \frac{9}{16}
 \end{array}$$



$$\begin{array}{r}
 (4) \quad \frac{12}{16} \\
 - \quad \frac{5}{6} \\
 \hline
 \end{array}$$



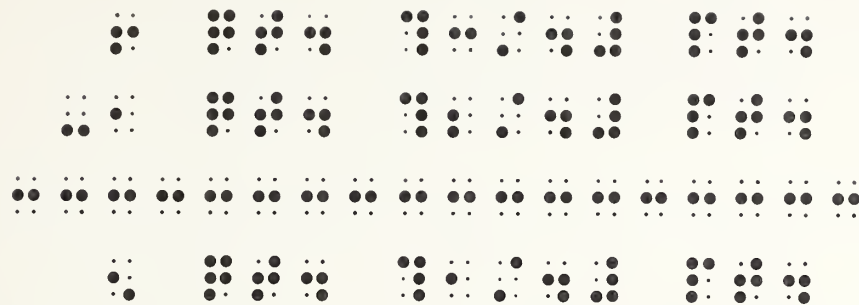
$$\begin{array}{r}
 (5) \quad \frac{1}{2} \text{ lb.} \\
 - \quad \frac{1}{2} \text{ lb.} \\
 \hline
 \end{array}$$



$$(6) \quad 6 \text{ qt. } \frac{3}{4} \text{ pt.}$$

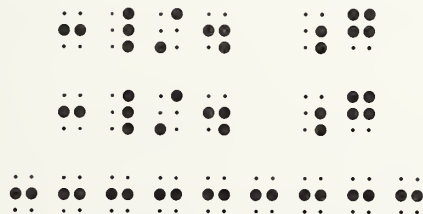
$$-1 \text{ qt. } \frac{2}{4} \text{ pt.}$$

$$5 \text{ qt. } \frac{1}{4} \text{ pt.}$$



$$(7) \quad \frac{3}{4} \text{ g}$$

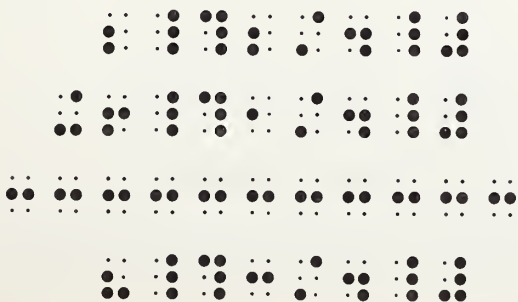
$$\frac{3}{4} \text{ g}$$



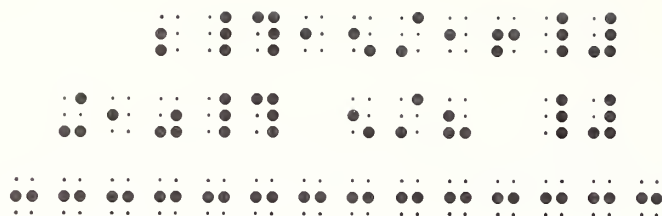
$$(8) \quad 2\frac{2}{4}$$

$$+6\frac{1}{4}$$

$$8\frac{3}{4}$$

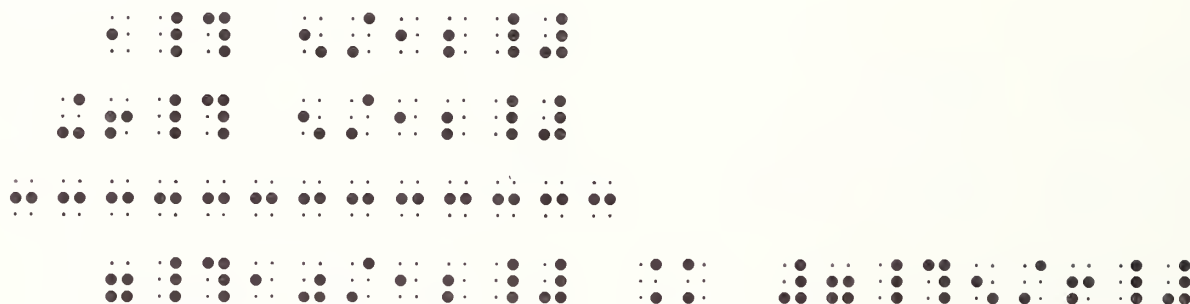


$$(9) \quad \begin{array}{r} 2\frac{15}{16} \\ + 10\frac{5}{8} \\ \hline \end{array}$$

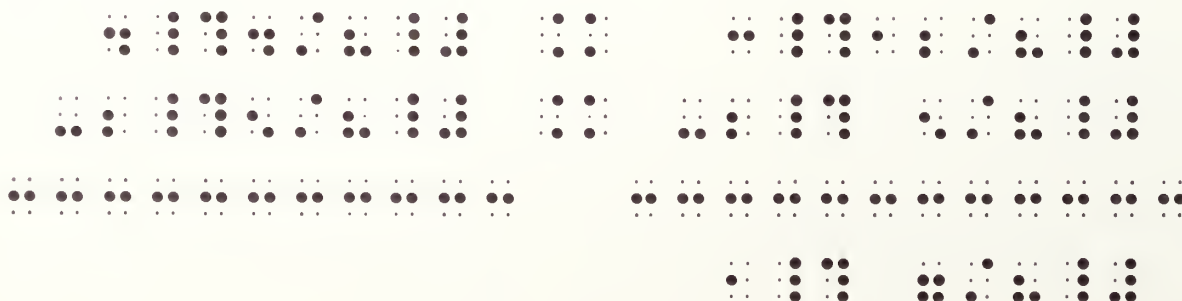


$$(10) \quad \begin{array}{r} 1\frac{5}{12} \\ + 6\frac{5}{12} \\ \hline \end{array}$$

$$7\frac{10}{12} = 7\frac{5}{6}$$



$$(11) \quad \begin{array}{r} 4\frac{4}{8} = 3\frac{12}{8} \\ - 2\frac{5}{8} = -2\frac{5}{8} \\ \hline 1\frac{7}{8} \end{array}$$



(12)

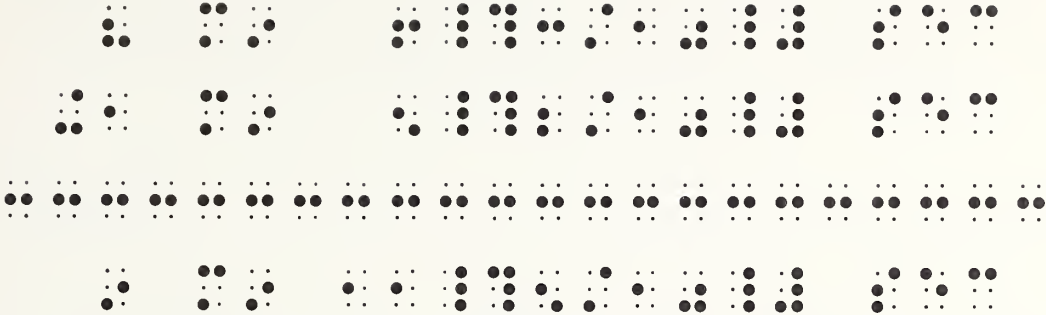
8 min

6³/₁₀ sec

+1 min

5²/₁₀ sec

9 min 11⁵/₁₀ sec

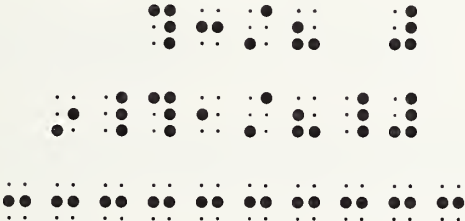


(13)

3

8

9¹/₈

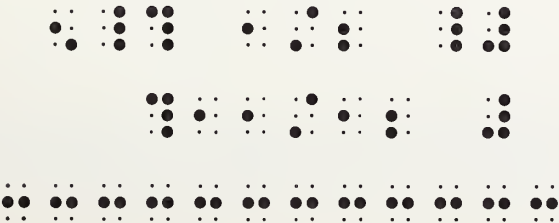


(14)

5¹/₂

11

12



(15) $\frac{216}{4}$

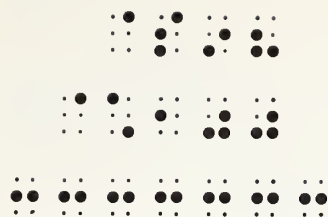
(16) $\frac{216}{4}$

(the fraction is shown directly beneath the whole number in print)

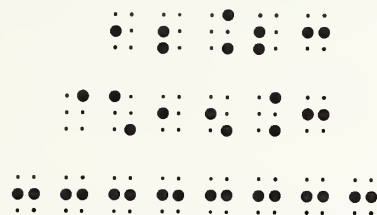
(17) $\frac{216}{4}$

g. In polynomials arranged spatially for addition or subtraction, each term, including its signs of operation, coefficients, letters, superscript, subscript, and baseline indicators, must be vertically aligned. When the baseline indicator is required, it must be placed in the first possible position consistent with this alignment. The corresponding symbols in each coefficient and superscript must be aligned vertically.

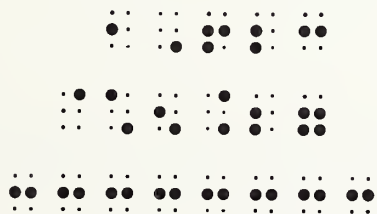
(4) $\begin{array}{r} \$98 \\ \times 100 \\ \hline \end{array}$



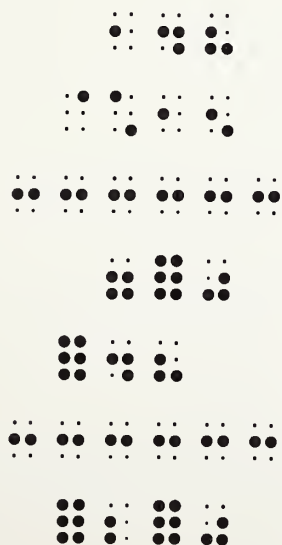
(5) $\begin{array}{r} 12.23 \\ \times 15.3 \\ \hline \end{array}$



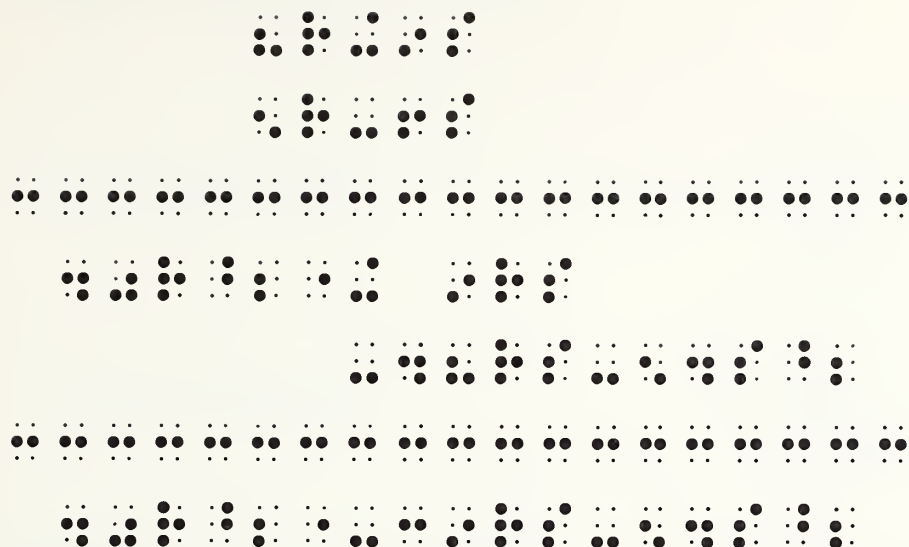
(6) $\begin{array}{r} 1,623 \\ \times 5.27 \\ \hline \end{array}$



(7) $\begin{array}{r} 148 \\ \times 15 \\ \hline 7?0 \\ ?48 \\ \hline ?2?0 \end{array}$

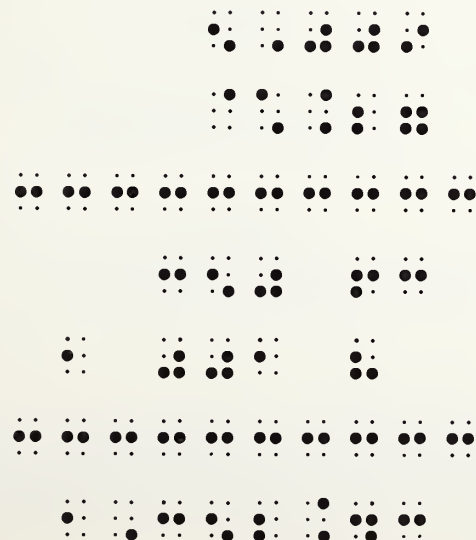


$$\begin{array}{r}
 (4) \quad 8r + 9s \\
 5r - 6s \\
 \hline
 40r^2 + 9rs \\
 -48rs - 54s^2 \\
 \hline
 40r^2 - 39rs - 54s^2
 \end{array}$$



b. If a comma or a decimal point appears in the answer, the corresponding cells in the partial products above it should be left blank.

$$\begin{array}{r}
 (1) \quad 5,009 \\
 \times .27 \\
 \hline
 35063 \\
 10018 \\
 \hline
 1,352.43
 \end{array}$$



c. In multiplication with subscripts denoting nondecimal bases, the partial products should be aligned for addition, and the subscript indicators should be vertically aligned immediately to the right of the addition arrangement.

(1)

54_{eight}

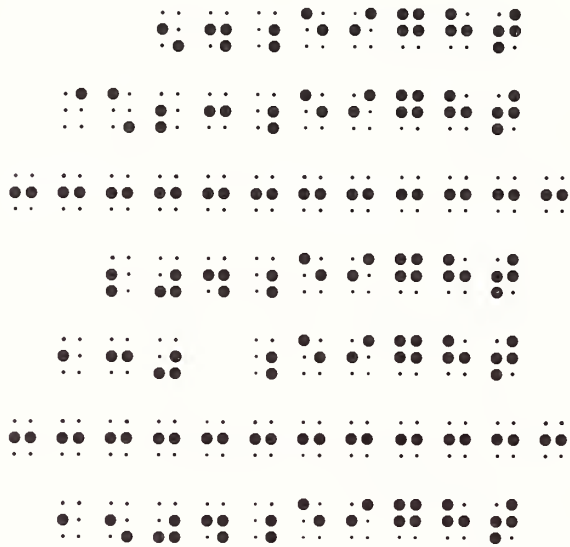
×

23_{eight}

204_{eight}

130_{eight}

1504_{eight}



Division Signs

Curved Division Sign on Left,
Separation Line Above



Curved Division Sign on Right,
Separation Line Above



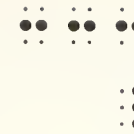
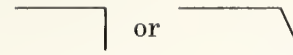
Curved Division Signs on Left and Right,
Separation Line Above



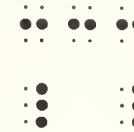
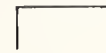
Straight or Slant Division Sign on Left,
Separation Line Above



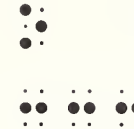
**Straight or Slant Division Sign on Right,
Separation Line Above**



**Straight Division Signs on Left and Right,
Separation Line Above**



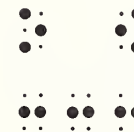
**Curved Division Sign on Left,
Separation Line Below**



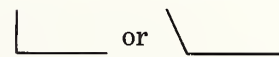
**Curved Division Sign on Right,
Separation Line Below**



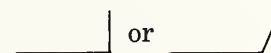
**Curved Division Signs on Left and Right,
Separation Line Below**



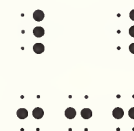
**Straight or Slant Division Sign on Left,
Separation Line Below**



**Straight or Slant Division Sign on Right,
Separation Line Below**



**Straight Division Signs on Left and Right,
Separation Line Below**



**Vertical Line for Division
Arrangements (varying in length)**

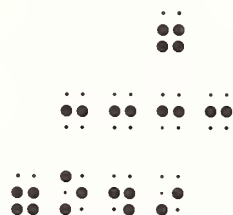


§164. Spatial Arrangement With Division: In spatial arrangements for division, the dividend and the partial products and differences must be aligned as in ink-print. The quotient must also be aligned with the dividend as in ink-print unless it has been intentionally misaligned as an exercise for the student.

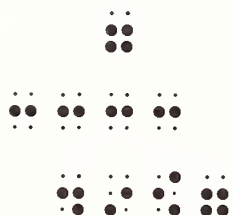
a. The appropriate division symbol ($\begin{smallmatrix} \bullet & \bullet \\ \vdots & \vdots \\ \bullet & \bullet \end{smallmatrix}$, $\begin{smallmatrix} \bullet & \bullet \\ \vdots & \vdots \\ \bullet & \bullet \end{smallmatrix}$, or $\begin{smallmatrix} \bullet & \bullet \\ \vdots & \vdots \\ \bullet & \bullet \end{smallmatrix}$) must be placed between the divisor and dividend as well as between the divisor and the quotient if they appear on the same line in ink-print. The horizontal line sometimes placed under the divisor as part of the ink-print sign should not be shown.

b. Each separation line (dots 2-5) must begin at the division sign and end at another division sign if it appears in ink-print. If the other division sign is not shown, each separation line must end one cell beyond the overall arrangement. All separation lines must be the same length.

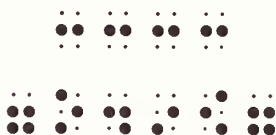
(1) $\begin{array}{r} 7 \\ 7 \overline{)49} \end{array}$



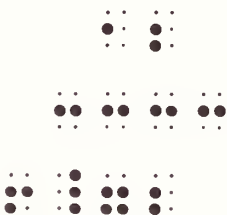
(2) $\begin{array}{r} 7 \\ 49 \overline{)7} \end{array}$



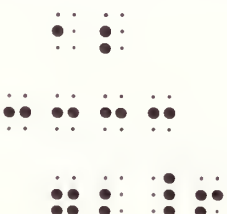
(3) $7 \overline{)49} 7$



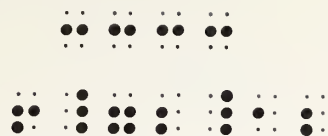
(4) $6 \overline{)72} 12$



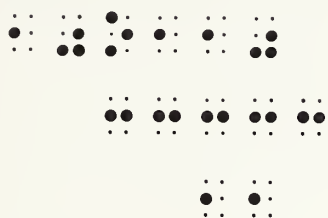
(5) $\overline{)72} 6$



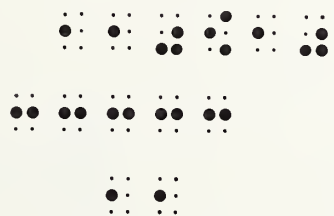
$$(6) \quad 6 \overline{72} 12$$



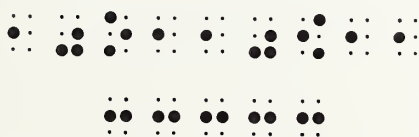
$$(7) \quad 10 \overline{110} 11$$



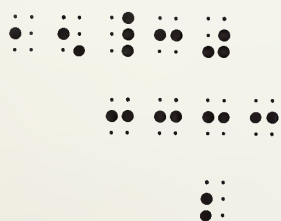
$$(8) \quad \underline{110} (10 \overline{11})$$



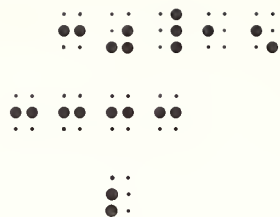
$$(9) \quad 10 \overline{110} (11)$$



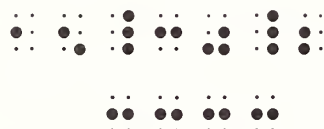
$$(10) \quad 15 \overline{30} 2$$



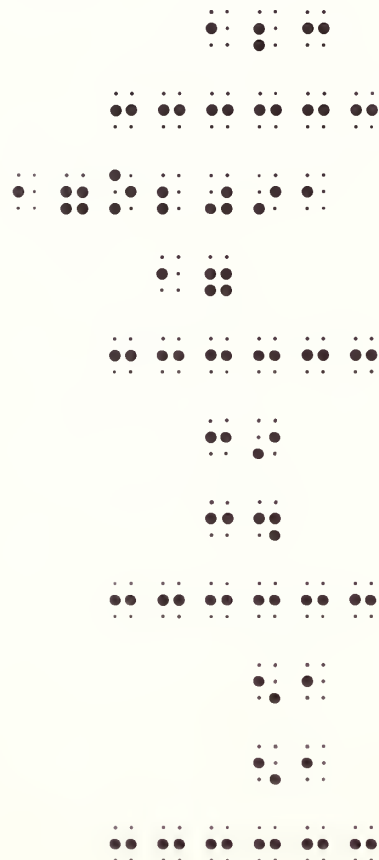
$$(11) \quad \begin{array}{r} 30 \overline{) 15} \\ 2 \end{array}$$



$$(12) \quad 15 \overline{) 30} \quad 2$$



$$(13) \quad \begin{array}{r} 123 \\ 17 \overline{) 2091} \\ \underline{17} \\ 39 \\ \underline{34} \\ 51 \\ \underline{51} \\ 00 \end{array}$$



(14)

x+ 4

x-3

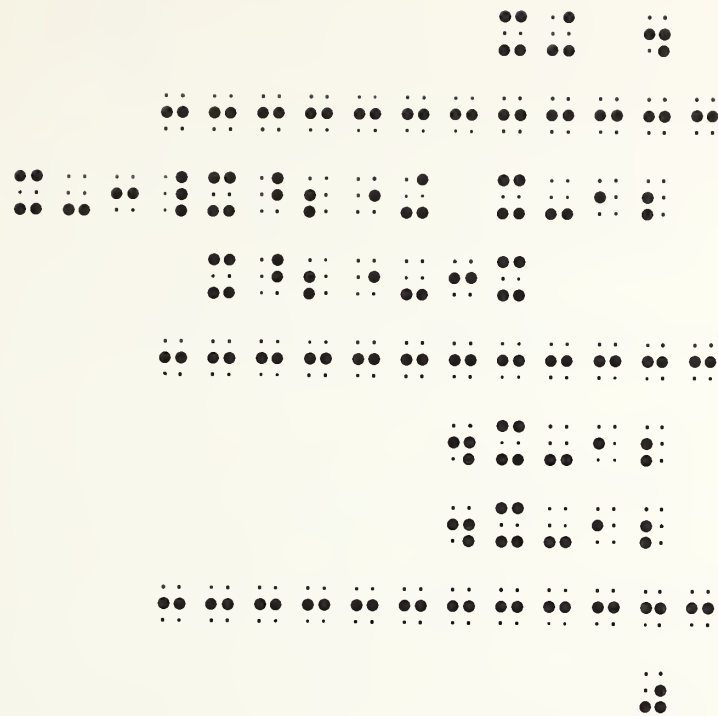
x²+ x-12

x²-3x

4x-12

4x-12

0



(15)

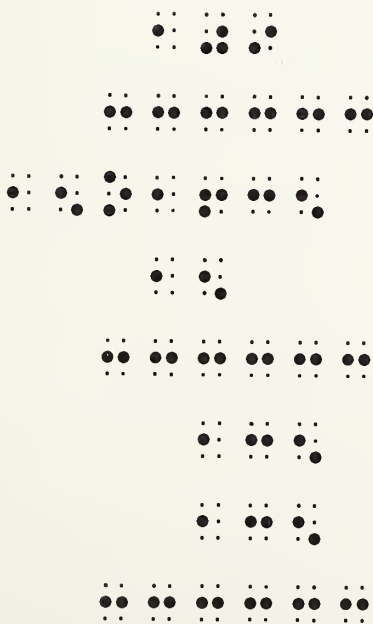
109

15)1635

15

135

135



(quotient unaligned in print)

(3)

2.5

11.4

28.7

25

0

37

25

120

100

20

The image displays a collection of Braille characters arranged in a grid-like pattern. The characters are organized into rows and columns, with some characters appearing in groups. The pattern is as follows:

- Row 1: 3 characters (each a 2-dot Braille character).
- Row 2: 10 characters (each a 2-dot Braille character).
- Row 3: 15 characters (each a 2-dot Braille character).
- Row 4: 2 characters (each a 2-dot Braille character).
- Row 5: 10 characters (each a 2-dot Braille character).
- Row 6: 2 characters (each a 2-dot Braille character).
- Row 7: 10 characters (each a 2-dot Braille character).
- Row 8: 3 characters (each a 2-dot Braille character).
- Row 9: 3 characters (each a 2-dot Braille character).
- Row 10: 10 characters (each a 2-dot Braille character).
- Row 11: 3 characters (each a 2-dot Braille character).
- Row 12: 10 characters (each a 2-dot Braille character).
- Row 13: 2 characters (each a 2-dot Braille character).

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d. The capitalized or uncapitalized “r”, indicating a remainder, should be preceded by a space and separated by the multipurpose indicator (dot 5) from the numeral following it.

$$\begin{array}{r} 18 \text{ r}2 \\ (1) \ 25 \overline{)452} \\ \underline{25} \\ 202 \\ \underline{200} \\ 2 \end{array}$$

A 10x10 grid of dots forming a sparse, symmetrical pattern. The pattern consists of several small clusters of dots arranged in a cross-like shape, with additional dots forming a larger, more complex structure in the center. The dots are black and the background is white.

$$\begin{array}{r} 25 \text{ R}12 \\ (2) \ 17 \overline{)437} \\ \underline{34} \\ 97 \\ \underline{85} \\ 12 \end{array}$$

A 10x10 grid of dots forming a sparse, abstract pattern. The dots are arranged in a way that suggests a larger, more complex structure, possibly a face or a figure, though the pattern is highly irregular and sparse.

e. A vertical line in a division arrangement may be either drawn or represented by dots 4-5-6. One space must be left between the vertical line and any digit preceding or following it.

$$\begin{array}{r} \text{(1)} \quad 7 \overline{) 539} \\ \underline{70} \\ 469 \\ \underline{140} \\ 329 \\ \underline{210} \\ 119 \\ \underline{119} \\ 0 \end{array}$$

The image displays a 10x10 grid of 100 small dot patterns. Each pattern is a unique combination of 10 binary states (on/off), arranged in a grid where each row and column contains all possible combinations of the 10 states. The patterns are arranged in a grid where each row and column contains all possible combinations of the 10 states, effectively creating a visual representation of a 10-bit binary space.

f. **Carried Numbers in Division:** When carried numbers appear in a division arrangement below their columns, the transcriber must insert the indicator for carried numbers between them and their related arrangement. The carried number indicator must have the same length as the separation line. Blank spaces must be left in the dividend to allow separate columns for the carried numbers.

$$(1) \begin{array}{r} 769 \\ 7 \overline{)53_4 8_6 3} \end{array}$$

g. If the division arrangement shows only a divisor and a dividend, and the divisor and dividend are composed entirely of numerals, the arrangement must not be regarded as spatial. It is not necessary to show the separation line or to leave a blank line above or below the arrangement. The numeric indicator must be used where required according to other rules of the code.

$$(1) \begin{array}{r} 3 \overline{)936} \end{array}$$

$$(2) \begin{array}{r} 312 \\ 3 \overline{)936} \end{array}$$

$$(3) \begin{array}{r} 2 \overline{)458} \end{array}$$

$$(4) \begin{array}{r} 2 \overline{)458(229} \end{array}$$

$$(5) \begin{array}{r} 2 \overline{)697 \text{ qt. } 7 \text{ pt.}} \end{array}$$

$$(6) \begin{array}{r} -4a^2 \overline{)16a^4 - 8a^3 - 24a^2} \end{array}$$

(7) You will notice a numeric relationship between the divisor and the dividend in

$$5 \overline{)20}$$

Square Root Sign

§165. Square Root Spatial Arrangements: A square root spatial arrangement should conform to the ink-print format, and the procedures used with division arrangements should be applied. The termination indicator is not required in a spatial square root arrangement.

$$(1) \quad \begin{array}{r} 6.48 \\ \sqrt{42.0000} \\ 36 \end{array}$$
$$\begin{array}{r} 124 \overline{) 600} \\ \times 4 \quad 496 \\ \hline 1288 \overline{) 10400} \\ \times 8 \quad 10304 \\ \hline 96 \end{array}$$

A 10x10 grid of dots forming a sparse, abstract pattern. The dots are arranged in a way that suggests a larger, more complex structure, possibly a face or a figure, though the pattern is highly irregular and sparse.

§166. Spatial Arrangement for Synthetic Division:

divisor	+2		1	−3	+4	+5	dividend
				+2	−2	+4	product
quotient			1	−1	+2	+9	remainder

In a synthetic division arrangement, the numerals in the dividend, product, and quotient must be aligned in vertical columns, as in ink-print. Signs of operation, if any, must also be vertically aligned. At least one blank cell must be left between adjacent columns.

A vertical line (dots 4-5-6) must be used between the synthetic divisor and the synthetic division arrangement in the position to the left or right as shown in ink-print. It must begin at the dividend and end at the product. No space should be left between the vertical line and the dividend or divisor. The separation line (dots 2-5) must extend from the vertical line to one cell beyond the synthetic arrangement. The vertical line should be used between the quotient and the remainder, as in ink-print. If the synthetic divisor appears boxed in on two sides, the boxing should be omitted.

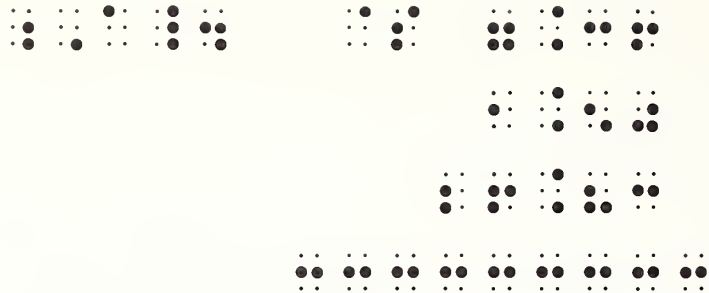
$$(1) \quad +2 \left| \begin{array}{ccc|c} 1 & -3 & +4 & +5 \\ & +2 & -2 & +4 \\ \hline & 1 & -1 & +2 \end{array} \right| +9$$

$$(2) \quad \begin{array}{ccc|c} 3 & -7 & -1 & -23 \\ & +9 & +6 & +15 \\ \hline 3 & +2 & +5 & -8 \end{array}$$

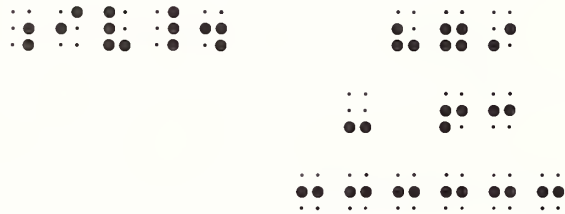
$$(3) \begin{array}{r} -1 \quad 1+2+2+4 \\ -1-1-1 \\ \hline 1+1+1 \mid +3 \end{array}$$

(In ink-print, the divisor is boxed in on two sides; there is no vertical line after the divisor.)

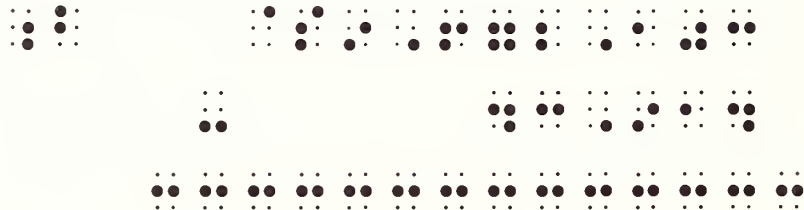
(3) A. \$ 7.36
 1.50
 26.83



(4) iv. 879
 — 63

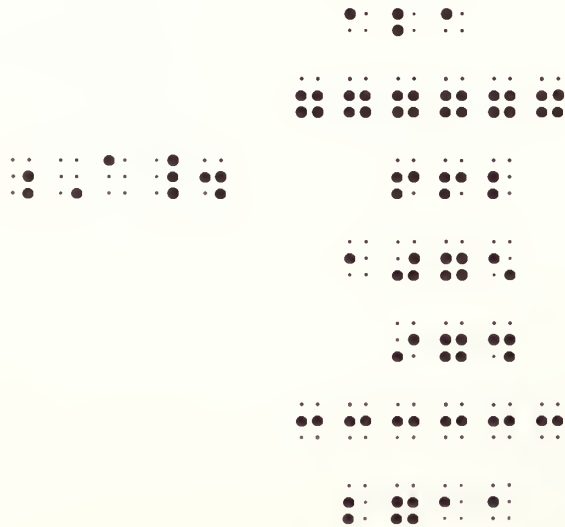


(5) b \$9,672,103
 — 43,914

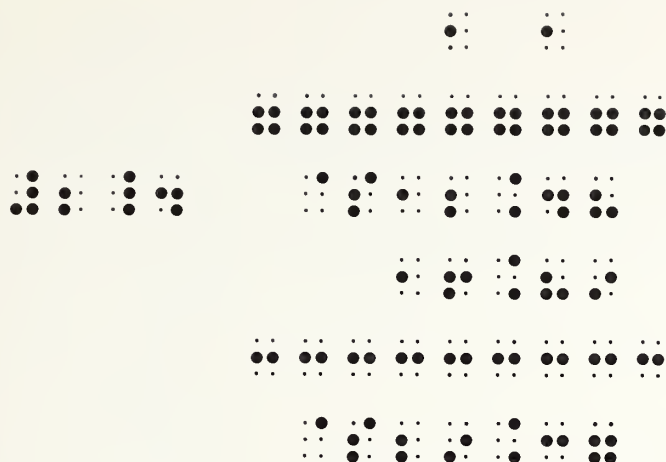


b. Addition With Carried Numbers: The identifier must be placed on the first line of the addition.

(1) A. ¹²¹662
 1075
 974
 2711

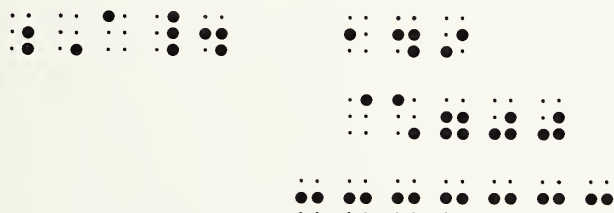


(2)	2.	\$12.48
		16.89
		<hr/>
		\$29.37

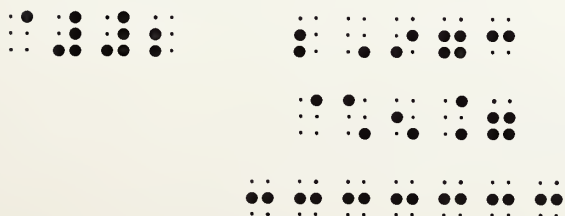


c. Multiplication: The identifier must be placed on the first line of the multiplication.

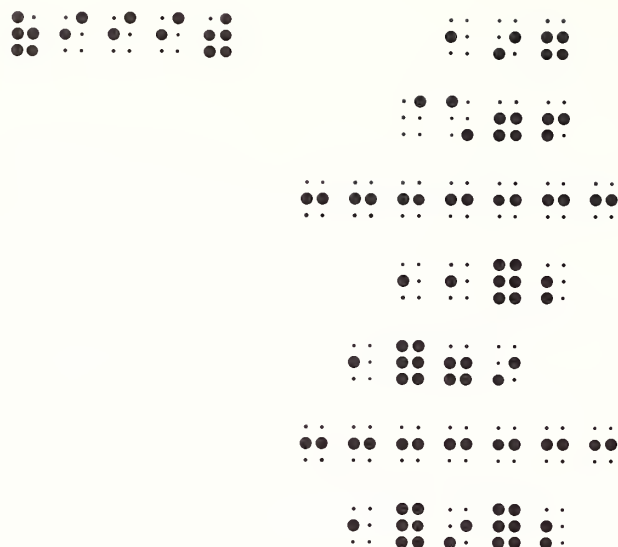
(1) A. 149×700



(2) $\begin{array}{r} *2 \quad 2,973 \\ \times 5.7 \\ \hline \end{array}$

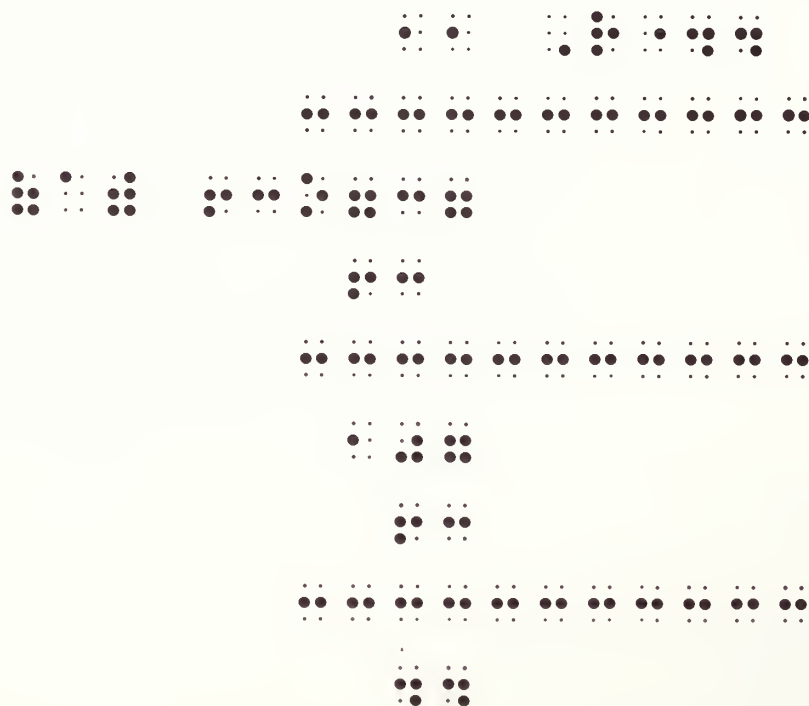


(3) (iii)
$$\begin{array}{r} 197 \\ \times 76 \\ \hline 11\ 72 \\ 1\ 79 \\ \hline 1\ 79\ 72 \end{array}$$



d. **Division:** The identifier must be placed on the line with the dividend.

(1) (a)
$$\begin{array}{r} 11\ R44 \\ 63 \overline{) 737} \\ \underline{63} \\ 107 \\ \underline{63} \\ 44 \end{array}$$



(2) B. $12\overline{)144}(12$

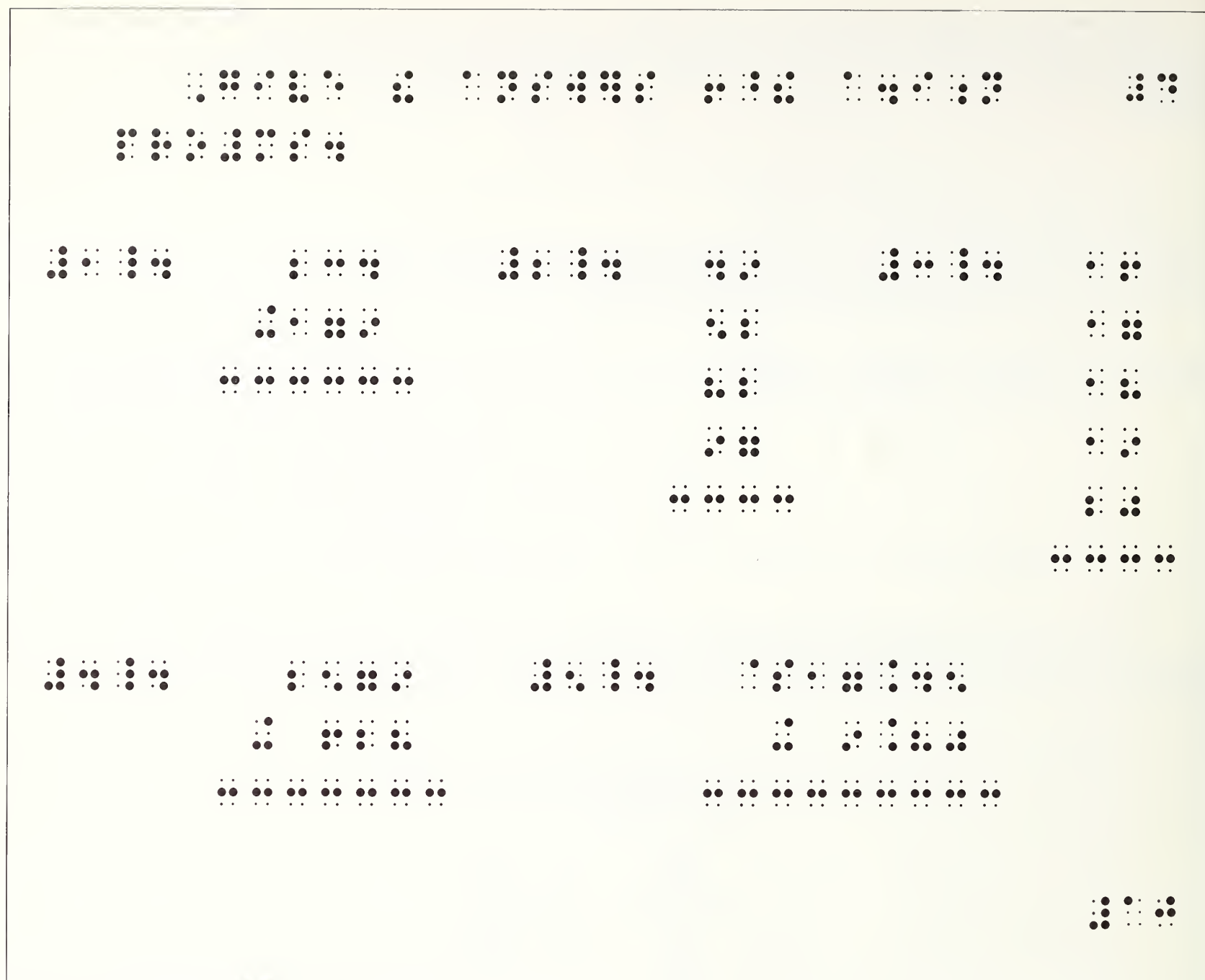
(3) 3. $72\overline{)109185}$

e. **Radicals:** The identifier must be placed on the line with the radicand.

(1) ☆4. $\sqrt[7]{5476}.$
 $\begin{array}{r} 49 \\ 144 \overline{) 576} \\ \underline{576} \end{array}$

f. **Synthetic Division:** The identifier must be placed on the line with the synthetic dividend.

(1) C. $\begin{array}{r|rr} 1 & -3 & 2 \\ & 2 & -2 \\ \hline 1 & -1 & 0 \end{array}$



HOMework

Prepare the following homework for submission to your teacher. Proofread carefully.

EXERCISE 15

$$\begin{array}{r}
 1. \quad 736 \\
 850 \\
 792 \\
 + 998 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 2. \quad 9104 \\
 6783 \\
 8860 \\
 + 569 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 3. \quad \$43.86 \\
 9.20 \\
 + 4.00 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 4. \quad \$.46 \\
 3.14 \\
 + 5.18 \\
 \hline
 \end{array}$$

$$\begin{array}{r} 5. \quad -130 \\ -112 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 867 \\ -46 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 12,134 \\ -8,359 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad \$602.09 \\ -537.40 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad \$156.24 \\ -95.07 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad 17x+2y+9z \\ -6x-5y+18z \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 4?1 \\ 25? \\ ?84 \\ \hline 900 \end{array}$$

$$\begin{array}{r} 12. \quad ?01? \\ -2?64 \\ \hline 62?6 \end{array}$$

$$\begin{array}{r} 13. \quad 3 \text{ ft. } 12 \text{ in.} \\ 4 \text{ ft. } 7 \text{ in.} \\ +3 \text{ ft. } 6 \text{ in.} \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 1 \text{ lb } 12 \text{ oz} \\ +2 \text{ lb } 8 \text{ oz} \\ \hline 3 \text{ lb } 20 \text{ oz} = 4 \text{ lb } 4 \text{ oz} \end{array}$$

$$\begin{array}{r} 15. \quad 14 \text{ hr. } 12 \text{ min.} \\ -2 \text{ hr. } 11 \text{ min.} \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 9 \text{ yr } 1 \text{ mo} = 8 \text{ yr } 13 \text{ mo} \\ -2 \text{ yr } 4 \text{ mo} = 2 \text{ yr } 4 \text{ mo} \\ \hline 6 \text{ yr } 9 \text{ mo} \end{array}$$

$$\begin{array}{r} 17. \quad \frac{1}{5} \\ +\frac{3}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad \frac{11}{12} \\ +\frac{5}{12} \\ \hline \end{array}$$

$$\begin{array}{r} 19. \quad \frac{3}{10} \\ +\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad \frac{1}{2} \text{ oz.} \\ -\frac{3}{8} \text{ oz.} \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad \frac{7}{8} \text{ in} \\ -\frac{5}{8} \text{ in} \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad 6\frac{5}{6} \\ +4\frac{5}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 23. \quad 10\frac{3}{16} \\ -3\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 24. \quad 10\frac{2}{3} \\ -\frac{7}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 25. \quad 16 \\ -\frac{1}{2} \\ \hline 15\frac{?}{2} \end{array}$$

$$\begin{array}{r} 26. \quad 7 \\ -\frac{5}{8} \\ \hline 6\frac{?}{8} \end{array}$$

$$\begin{array}{r} 27. \quad 9 \text{ min } 4\frac{1}{2} \text{ sec} \\ -2 \text{ min } 7 \text{ sec} \\ \hline \end{array}$$

$$\begin{array}{r} 28. \quad +5c^2-4cd+3d^2 \\ -4c^2+3cd \\ -c^2 \quad -2d^2 \\ \hline -cd+d^2 \end{array}$$

$$\begin{array}{r} 29. \quad \begin{array}{r} 11 \\ 760 \\ 333 \\ +989 \end{array} \\ \hline 2082 \end{array}$$

$$\begin{array}{r} 30. \quad 453 \\ \times ? \\ \hline 906 \end{array}$$

$$\begin{array}{r} 31. \quad \$8.56 \\ \times 670 \\ \hline \end{array}$$

$$\begin{array}{r} 32. \quad \$6.50 \\ \times .07 \\ \hline \end{array}$$

$$\begin{array}{r} 33. \quad 74.12 \\ \times 8.3 \\ \hline 22236 \\ 59296 \\ \hline 615.196 \end{array}$$

$$\begin{array}{r} 34. \quad 486 \\ \times 34 \\ \hline 1944 \\ 14580 \\ \hline 16,524 \end{array}$$

$$\begin{array}{r} 35. \quad 1\frac{1}{3} \\ \times \frac{9}{10} \\ \hline \end{array}$$

$$\begin{array}{r} 36. \quad \frac{1}{2} \\ \times \frac{5}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 37. \quad 4\frac{1}{4} \\ \times 7 \\ \hline 3 \\ 1\frac{1}{4} \\ 28 \\ \hline 29\frac{3}{4} \end{array}$$

$$\begin{array}{r} 38. \quad 2x^2-3x+1 \\ \quad 3x-1 \\ \hline 6x^3-9x^2+3x \\ -2x^2+3x-1 \\ \hline 6x^3-11x^2+6x-1 \end{array}$$

$$\begin{array}{r} 39. \quad 604_{\text{seven}} \\ \times 36_{\text{seven}} \\ \hline 5133_{\text{seven}} \\ 2415_{\text{seven}} \\ \hline 32313_{\text{seven}} \end{array}$$

Complete the work, if necessary, and check the answers.

$$40. \begin{array}{r} 631 \text{ r}3 \\ 6 \overline{)3??9} \end{array}$$

$$41. \begin{array}{r} 5?4 \text{ r}1 \\ 8 \overline{)4033} \end{array}$$

$$42. \begin{array}{r} 41 \\ 4 \overline{)164} \\ 16 \\ \hline 4 \\ 4 \\ \hline \end{array}$$

$$43. \begin{array}{r} .47 \\ 15 \overline{)7.09} \\ 60 \\ \hline 109 \\ 105 \\ \hline 4 \end{array}$$

$$44. \begin{array}{r} 30.4 \\ .04 \overline{)1.216} \\ 12 \\ \hline 16 \\ 16 \\ \hline \end{array}$$

$$45. 3 \overline{)5 \text{ gal. 1 qt.}}$$

$$46. 19 \overline{)437(23)}$$

$$47. \begin{array}{r} x^2+3x-5 \\ 4x-3 \overline{)4x^3+9x^2+29x+17} \\ 4x^3+3x^2 \\ \hline 12x^2-29x+17 \\ 12x^2-9x \\ \hline -20x+17 \\ -20x+15 \\ \hline +2 \end{array}$$

$$48. \begin{array}{r} \$.21 \\ 15 \overline{)\$3.25} \\ 30 \\ \hline 25 \\ 15 \\ \hline 10 \end{array}$$

$$49. 5 \overline{)27^9 45}$$

Divide.

$$50. 36 \overline{)178}$$

$$51. 45 \overline{)9238}$$

$$52. 44 \overline{)42271}$$

$$53. 15 \overline{)14415}$$

$$54. \begin{array}{r} 5.74 \\ \sqrt{33.0000} \\ 25 \\ \hline 107 \overline{)800} \\ \times 7 \overline{)749} \\ 1144 \overline{)5100} \\ \times 4 \overline{)4576} \\ 524 \end{array}$$

$$55. \begin{array}{r} -3 \overline{)20-9173} \\ -618-2730 \\ \hline 2-69-1033 \end{array}$$

$$56. \begin{array}{r} 5-2-31 \overline{)2} \\ 101626 \\ \hline 5813 \overline{)27} \end{array}$$

57. Name the sums and missing addends.

$$\begin{array}{r} 5 \\ +5 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +5 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +6 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +2 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ -4 \\ \hline \end{array}$$


$$\begin{array}{r} 10 \\ -2 \\ \hline \end{array}$$


$$\begin{array}{r} 6 \\ +4 \\ \hline \end{array}$$

LESSON 16

SPATIAL ARRANGEMENTS (CONTINUED)

Cancellation Indicators

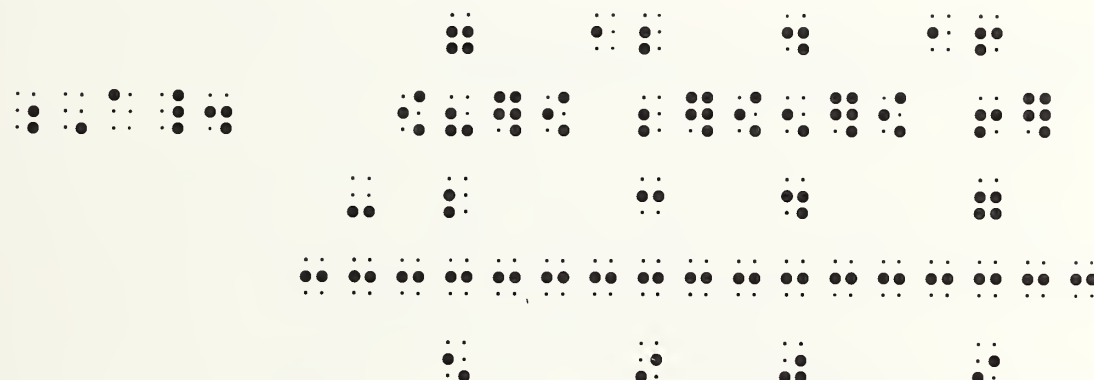
Opening 

Closing 

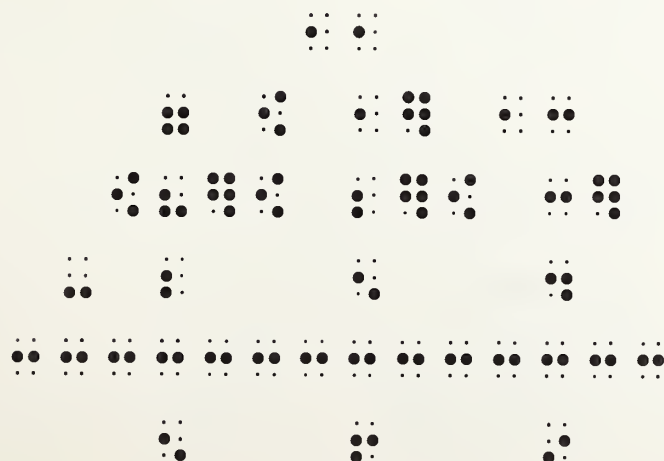
§173. **Cancellation and Spatial Arrangements:** A spatial arrangement must be used whenever numbers and letters are canceled in print by any type of stroke through them. The opening and closing cancellation indicators must enclose the material being canceled.

§174. **Cancellation in Subtraction:** When cancellation is shown in subtraction, the material should be aligned for computation, and spaces should be left, where necessary, to achieve this. An identifying numeral or letter should be placed on the line with the minuend.

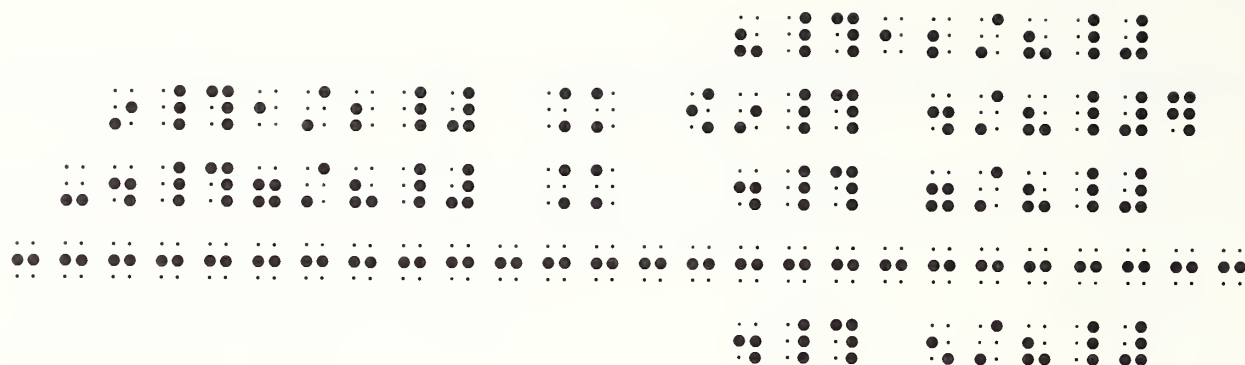
(1) A.
$$\begin{array}{r} \overset{7}{\cancel{8}} \overset{12}{\cancel{2}} \overset{4}{\cancel{5}} \overset{16}{\cancel{6}} \\ - 2 \ 3 \ 4 \ 7 \\ \hline 5 \ 9 \ 0 \ 9 \end{array}$$



(2)
$$\begin{array}{r} \overset{11}{\cancel{7}} \overset{13}{\cancel{2}} \overset{13}{\cancel{5}} \\ - 2 \ 5 \ 4 \\ \hline 5 \ 6 \ 9 \end{array}$$

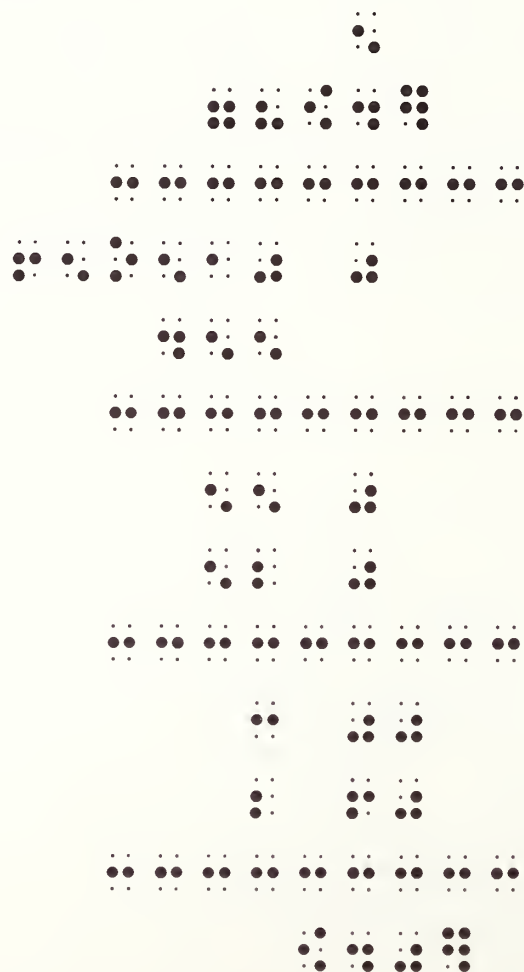


$$\begin{array}{r}
 8 \frac{12}{8} \\
 (3) \quad 9 \frac{1}{2} = \cancel{9 \frac{4}{8}} \\
 -4 \frac{7}{8} = 4 \frac{7}{8} \\
 \hline
 4 \frac{5}{8}
 \end{array}$$



§175. **Cancellation in Long Division:** If cancellation is shown in long division, the canceled material must be enclosed in cancellation indicators, and blank cells must be left, where necessary, for proper alignment.

$$\begin{array}{r}
 78 \cancel{4}^5 \\
 (1) \quad 65 \overline{) 5100} \\
 \underline{455} \\
 550 \\
 \underline{520} \\
 300 \\
 \underline{260} \\
 \underline{40}
 \end{array}$$

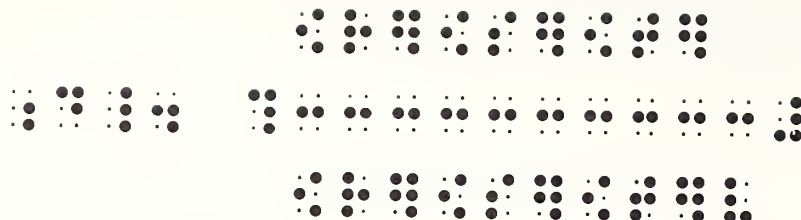


$$(1) \quad \frac{\frac{1}{\cancel{36}}}{6} = \frac{1}{6}$$

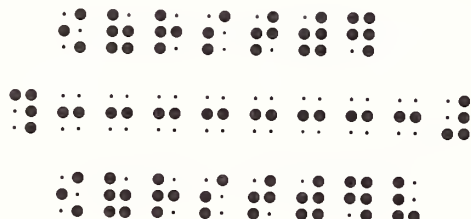
$$(2) \quad \frac{1}{2} \times \frac{8}{1} = \frac{1 \times 8}{1} = 8$$

(3)
$$\begin{array}{r} 15 \times 5280 \\ \hline 60 \times 60 \\ \hline \end{array}$$

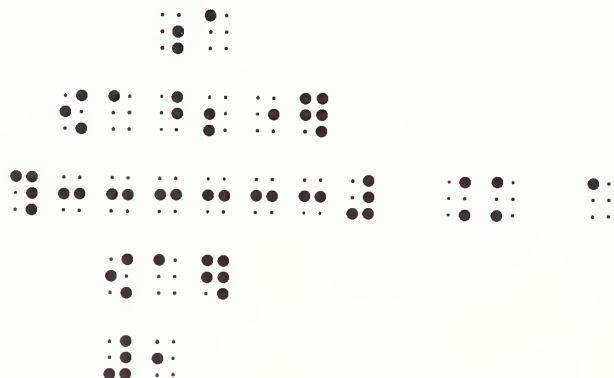
$$(4) \quad d. \frac{r's\cancel{v}}{r's\cancel{v}}$$



$$(5) \quad \frac{(rst)}{(rst) v}$$

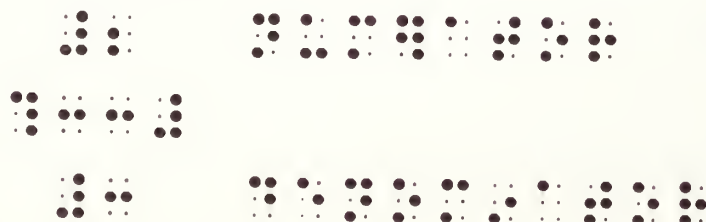


$$(6) \quad \frac{\overset{a}{\cancel{a}}}{\underset{1}{a}} = a$$

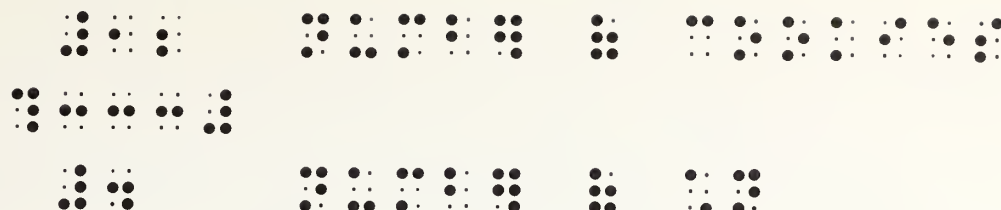


§177. **Simple Fractions Arranged Spatially for Illustration:** It is often helpful to use a spatial arrangement if the parts of a simple fraction are explained or identified. The numeric and English letter indicator must be used, where necessary, within the arrangement.

$$(1) \quad \frac{2 \text{ numerator}}{3 \text{ denominator}}$$



(2) $\frac{12}{4}$ number of cookies
number of children



§178. Hypercomplex Fractions:

Hypercomplex Fraction Indicators

Opening



Closing



Horizontal Hypercomplex Fraction Line



a. A hypercomplex fraction is one whose numerator or denominator, or both, contain at least one complex fraction. A fraction is not a hypercomplex fraction if the only complex fractions it contains are at the superscript or subscript level.

(1) $\frac{1}{\frac{\frac{a}{b}}{2^{\frac{c}{d}}}}$

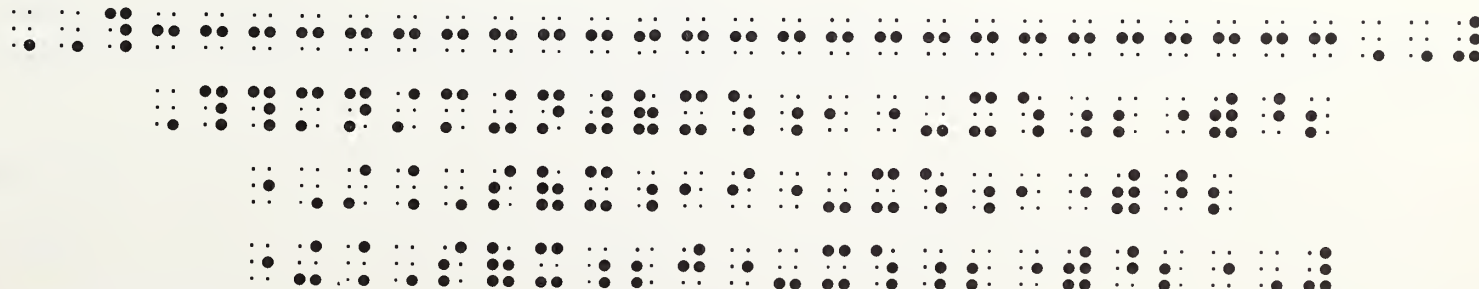
(this is not a hypercomplex fraction)

(2) $\frac{\frac{1}{2}}{\frac{\frac{3}{4}}{5}}$

(this is a hypercomplex fraction)

b. Hypercomplex fraction indicators must be used with hypercomplex fractions. Although it is permissible to use an entirely spatial arrangement or an entirely linear arrangement, it is preferable to use a linear arrangement within a spatial arrangement.

(1)
$$\frac{1}{\frac{\frac{mn}{m+n} (\bar{x}_1 - \bar{x}_2)^2}{\sum (x_{1i} - \bar{x}_1)^2 + \sum (x_{2j} - \bar{x}_2)^2}}$$



(preferred method)

$$(2) \quad \frac{1}{\frac{\frac{mn}{m+n} (\bar{x}_1 - \bar{x}_2)^2}{\sum (x_{1i} - \bar{x}_1)^2 + \sum (x_{2j} - \bar{x}_2)^2}}$$

(complete linear arrangement)

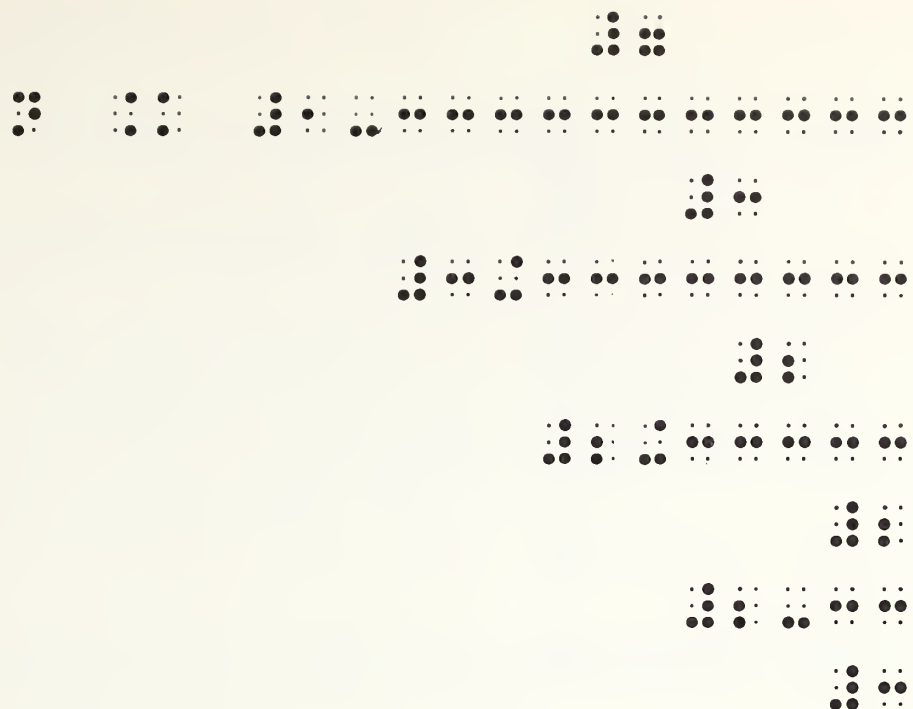
$$(3) \quad \frac{1}{\frac{\frac{mn}{m+n} (\bar{x}_1 - \bar{x}_2)^2}{(x_{1i} - \bar{x}_1)}}$$

(complete spatial arrangement)

Note: Hypercomplex fractions of higher order may be transcribed in the manner described above. Dot 6 should be added the appropriate number of times before the fraction indicators and the matching fraction lines.


§179. Continued Fractions: A continued fraction is one in which each denominator is the sum of a whole number and a fraction. Such a fraction must be transcribed entirely in a spatial arrangement. Each fraction line should be proportionately the same length shown in print. No fraction indicators may be used within a continued fraction.


$$(1) \quad n = 1 - \frac{7}{3 + \frac{3}{2 + \frac{2}{2 - \frac{2}{3}}}}$$



§180. **Enlarged Signs of Grouping:** Enlarged signs of grouping are used with determinants and matrices as well as with unified systems of equations.

Enlarged Parentheses

Left (


Right) 


Enlarged Brackets

Left [


Right] 


Enlarged Braces

Left { 

Right } 

Enlarged Vertical Bar

Single | 

Double || 

Enlarged Angle Brackets

Left	<	
Right	>	

Enlarged Barred Brackets

Left		
Right		

Enlarged Barred Braces

Left		
Right		

Enlarged Half Brackets

Upper Left	or	
Upper Right	or	
Lower Left	or	
Lower Right	or	

§181. Unified Systems of Equations:

a. When mathematical equations are arranged on two or more lines and joined by a sign of grouping, the arrangement is called a *unified system of equations*. Such an arrangement is considered to be spatial, and blank lines must be left above and below it. In braille, enlarged signs of grouping are used on each line of the unified system of equations and must be vertically aligned. The opening and the closing enlarged grouping symbol must be placed in the cells next to the items which extend furthest left and furthest right. If only the opening or closing sign of grouping is shown in print, only that sign should be shown in braille. Any material, such as punctuation, signs of comparison, signs of operation, or identifiers, should be shown on the top line in braille, even though it is centered in print.

(1) A. $\begin{cases} x = y \\ 5x - y = 4 \end{cases}$

(2) $\begin{cases} \frac{1}{2}x + y = 7 \\ 3x - 2y = 9 \end{cases}$

$$(3) \begin{cases} 0.5(2x+y) = -3 \\ 5y+x = 6 \end{cases}$$

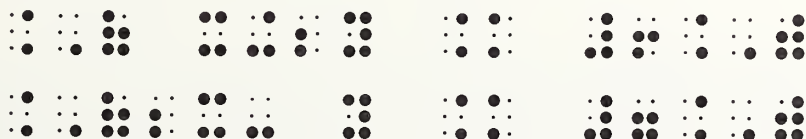


$$(4) \begin{cases} \frac{x}{3} - \frac{y}{2} = 2 \\ 5x+3y = 51 \end{cases} = (? , ?)$$



b. In a unified system of equations, the numeric indicator must be used or must not be used according to the rules of the code.

$$(1) \begin{cases} x+2y = 6 \\ 2x-y = 7 \end{cases}$$



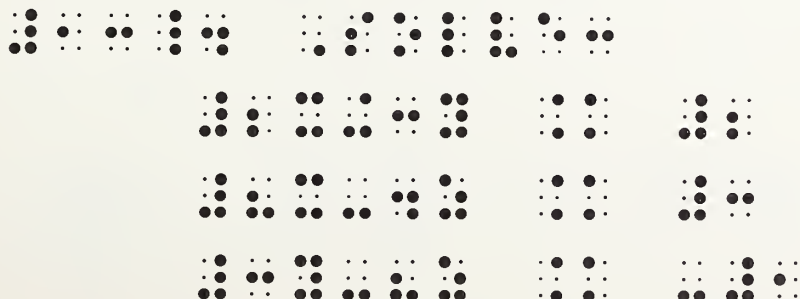
$$(2) \begin{cases} x+0y = 4 \\ 0x+y = 1 \end{cases}$$



c. **Ununified Systems of Equations:** When mathematical equations are arranged on two or more lines and are *not* joined by any sign of grouping, the arrangement is not considered to be spatial, and no blank lines need be left above or below it.

(1) 13. Solve:

$$\begin{aligned} 2x+3y &= 2 \\ 8x-4z &= 3 \\ 3y-8z &= -1 \end{aligned}$$

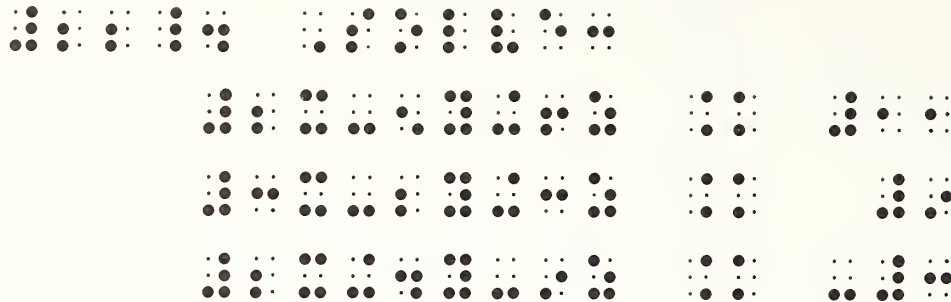


(2) 22. Solve:

$$2x - 5y + 6z = 11$$

$$3x - 2y + 3z = 9$$

$$2x + 4y - 9z = -3$$



§182. Enlarged Transcriber's Grouping Signs:

Left

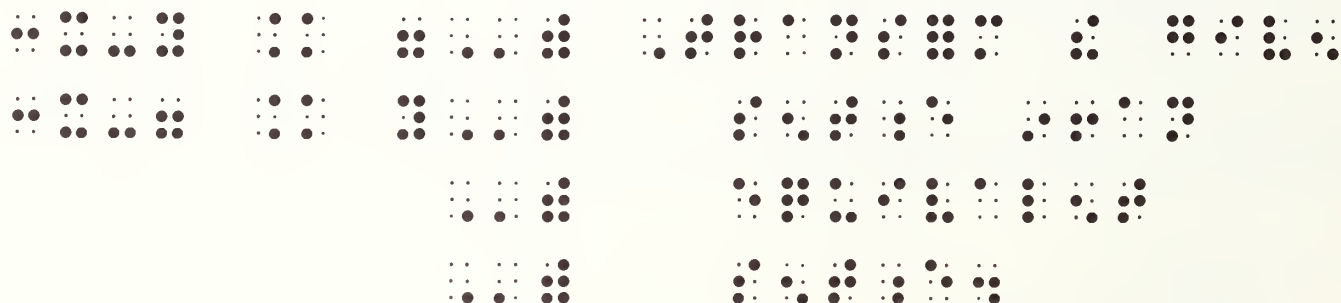
Right

When an explanation or comment refers to more than one ink-print line and no print grouping symbol links these lines, the implied grouping is shown by a transcriber's enlarged grouping sign. The left- or right-enlarged transcriber's grouping sign must be used according to the position of the explanation in print. There must be a blank space between the grouping symbol and the explanation. All runovers of the explanation must be indented two cells from the beginning of the first line of the explanation.

(1) $3x - y = 7$

Transform the given sentence
into an equivalent sentence.

$$3x - 7 = y$$



(2) $y > 2$

$x \in \mathcal{R}$ and

$y \in \mathcal{R}$

$x > 3$



§183. Determinants and Matrices:

a. Determinants and matrices are spatial arrangements. Thus, a line must be skipped above and below each determinant or matrix. At least one enlarged sign of grouping must appear on each line of the arrangement.

b. Each entry must be moved as far left as possible in its column, and one column of blank cells must be left between the columns of the arrangement.

c. Each opening sign of grouping must be in direct contact with an entry in the determinant or matrix. In the same way, at least one closing sign of grouping must appear in direct contact with an entry in the determinant or matrix.

d. The numeric indicator must be used with numeric entries in a determinant or matrix, even when such entries are in direct contact with an opening grouping sign.

$$(1) \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

$$\begin{array}{cc} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \\ \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \end{array}$$

$$(2) \left(\begin{array}{ccc} 1 & -\frac{4}{3} & \frac{5}{3} \\ 2 & 5 & 12 \end{array} \right)$$

$$\begin{array}{cc} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \\ \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \end{array}$$

e. The English letter indicator must not be used with any letter or combination of letters in a determinant or matrix.

$$(1) \left(\begin{array}{ccc} a & b & c \\ 0 & 0 & 0 \end{array} \right)$$

$$\begin{array}{cc} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \\ \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \end{array}$$

$$(2) \begin{vmatrix} ab & cd \\ ac & ce \end{vmatrix}$$

$$\begin{array}{cc} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \\ \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} \end{array}$$

f. Material outside the determinant or matrix, such as identifiers, punctuation, signs of operation, or signs of comparison, should be placed on the top line of the arrangement, even though it is centered in print.

$$(1) \quad 1. \quad \begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 4 \\ -5 \\ 0 \end{vmatrix}.$$

g. When dots are shown in a determinant or matrix to indicate omission of one or more rows, a series of dots 3 is used in braille.

i. If at least one dot appears in each column and no dots appear between columns, an ellipsis (...) is placed as far left as possible in each column.

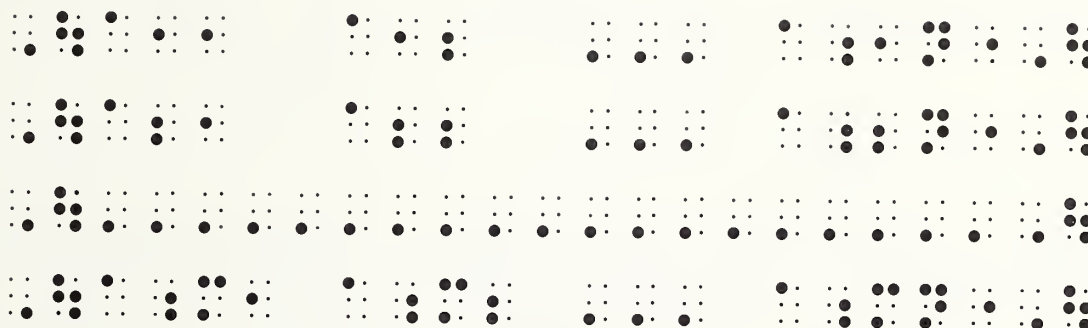
$$(1) \quad \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \dots & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

ii. When dots appear completely across a row and occupy space between the columns as well as in the columns, a sequence of dots 3 must be used, beginning in the first cell of the first column and extending to the end of the longest entry in the last column.

$$(1) \quad \left[\begin{array}{cccc} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \dots & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{array} \right]$$

iii. If some of the columns contain no dots, a line of dots 3 must be used as in ii above.

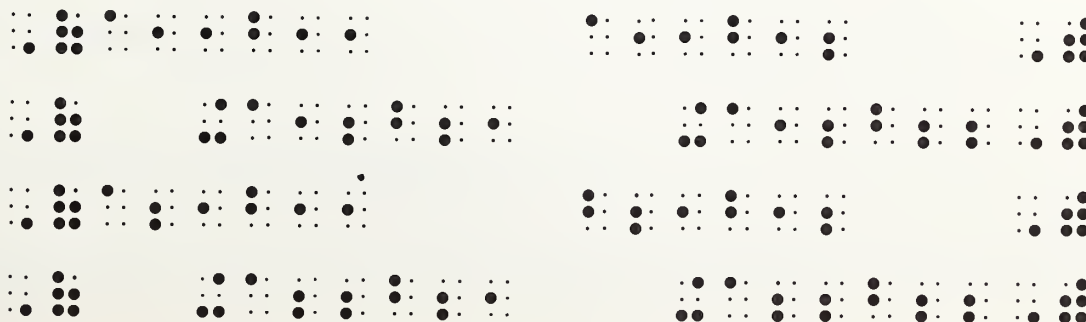
$$(1) \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{vmatrix}$$



h. Sometimes space-saving techniques must be adopted to confine the arrangement to one braille page.

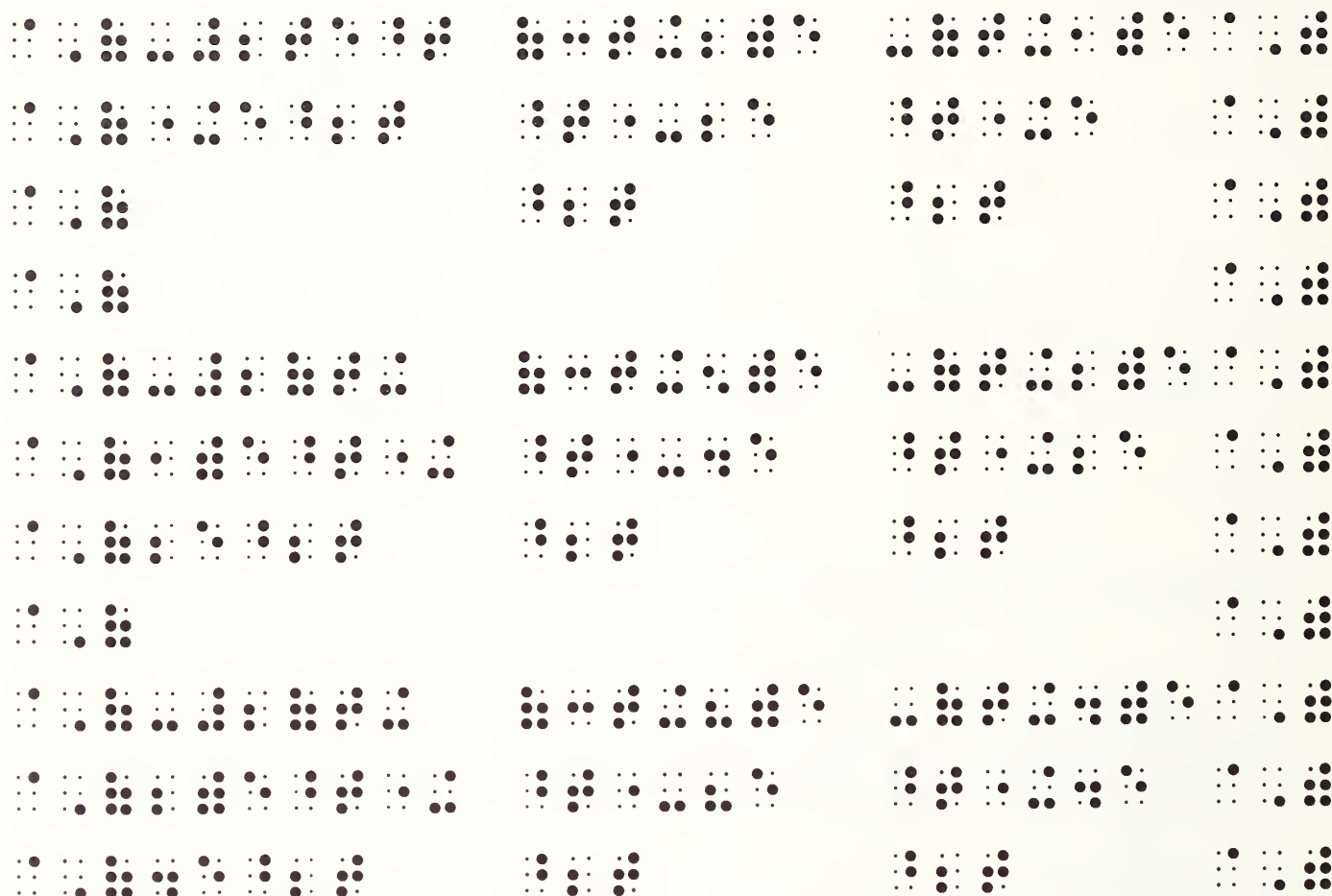
i. Entries may be run over to new lines. Each runover should be indented two cells from the first cell of the first line of the entry. No lines should be skipped between entries. If necessary, rules for preference in runovers can be ignored. No baseline indicator should be used before a closing grouping symbol unless it is the end of the item and touches the grouping symbol. This is the preferred space-saving technique.

$$(1) \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & b_{21}b_{12}+a_{22}b_{22} \end{pmatrix}$$



ii. Entries may be run over to new lines with no indentations. In this case, a line should be skipped between each row in the arrangement. Preference rules for runovers need not be observed if space would be saved.

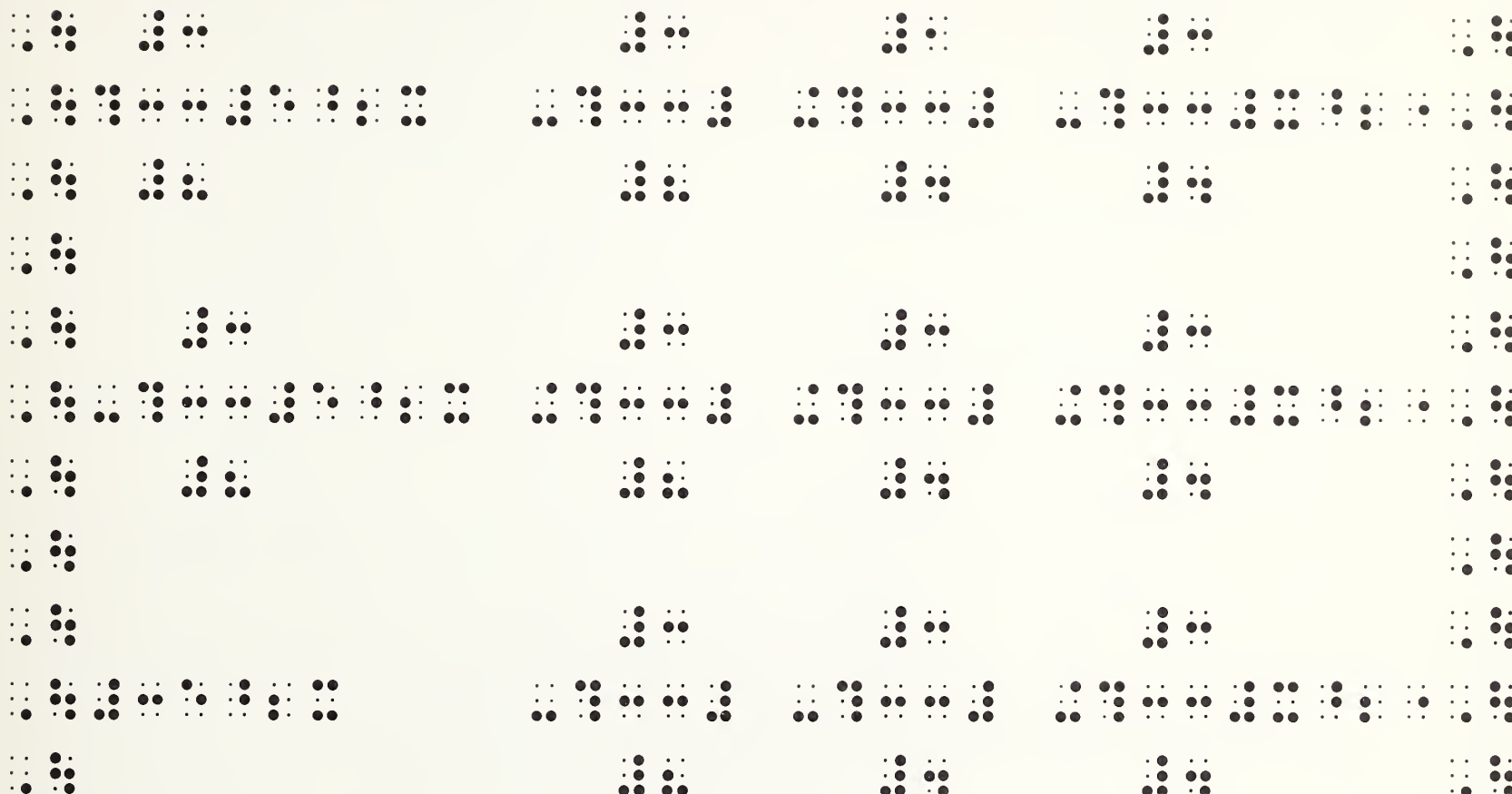
$$(1) \left[\begin{array}{ccc} -2te^t + e^{2t} & (3t+2)e^t - 2e^{2t} & -(t+1)e^t + e^{2t} \\ -2(t+1)e^t + 2e^{2t} & (3t+5)e^t - 4e^{2t} & -(t+2)e^t + 2e^{2t} \\ -2(t+2)e^t + 4e^{2t} & (3t+8)e^t - 8e^{2t} & -(t+4)e^t + 4e^{2t} \end{array} \right]$$



iii. Grouping symbols may be drawn in place of the braille equivalents.

iv. Fractions may be shown spatially. In this case, lines should be skipped above and below the rows containing the fraction.

$$(1) \left| \begin{array}{cccc} \frac{3}{8}e^{2x} & -\frac{3}{8} & +\frac{1}{4} & -\frac{3}{4}x^2 \\ -\frac{3}{8}e^{2x} & +\frac{3}{8} & +\frac{3}{4} & +\frac{3}{4}x^2 \\ 3e^{2x} & -\frac{3}{8} & -\frac{3}{4} & +\frac{3}{4}x^2 \end{array} \right|$$



v. When no other method saves the required space, the technique of keying, discussed in §186, should be used.

TABLES

§184. **Tables:** In transcribing tables, the rules provided in the *Code of Braille Textbook Formats and Techniques* should be followed.

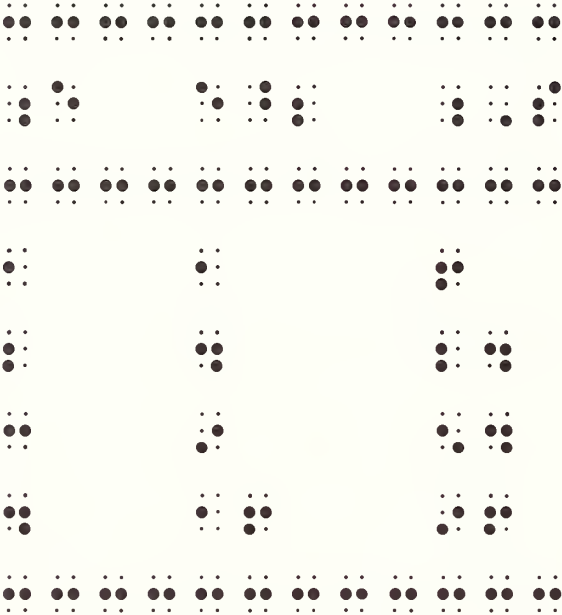
a. **Letters in Tables:** When letters appear in tables, whether as entries or headings, the English letter indicator must be used or must not be used as though the letters were not part of the table.

b. **Numbers in Table Headings:** When numerals appear in table headings, the rules for the use and nonuse of the numeric indicator must be followed.

c. **Numbers in Table Entries:** The numeric indicator must not be used when the entries in a table consist entirely of numerals, including interior commas and decimal points. If the entries in a table contain words, letters, signs of operation, signs of comparison, or any other mathematical signs, the numeric indicator must be used throughout the table.

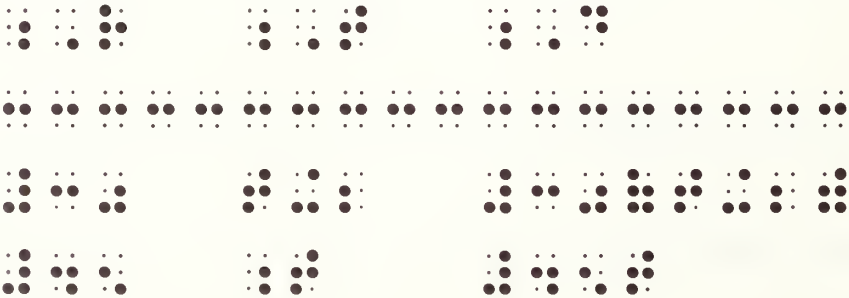
(1)

e	e ²	S
1	1	6
2	4	24
3	9	54
4	16	96



(2)

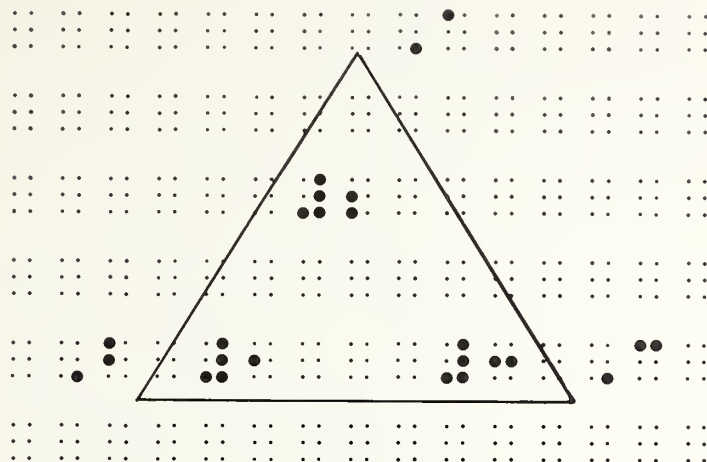
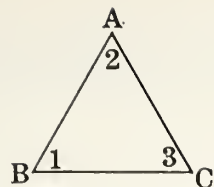
R	T	D
30	t+2	30(t+2)
45	t	45t



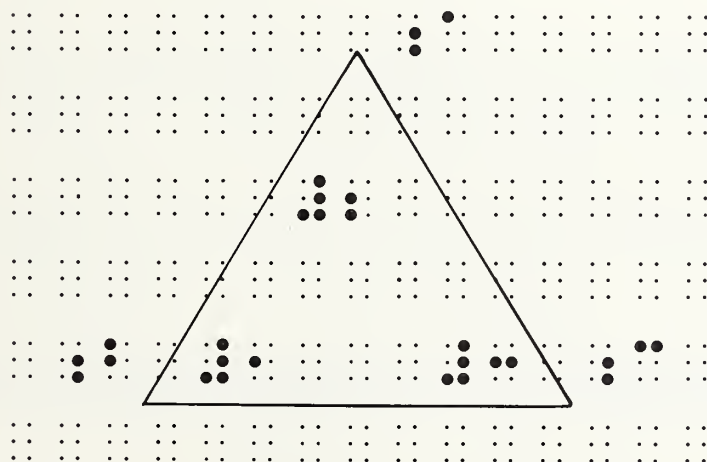
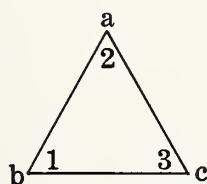
LABELS FOR FIGURES AND DIAGRAMS

§185. **Labels for Figures and Diagrams:** In labeling diagrams, the numeric indicator must be used with numerals. When a single English letter in regular type is used as a label in a diagram, the English letter indicator is required if the letter is in lower case, but omitted if the letter is capitalized.

(1)



(2)



KEYING

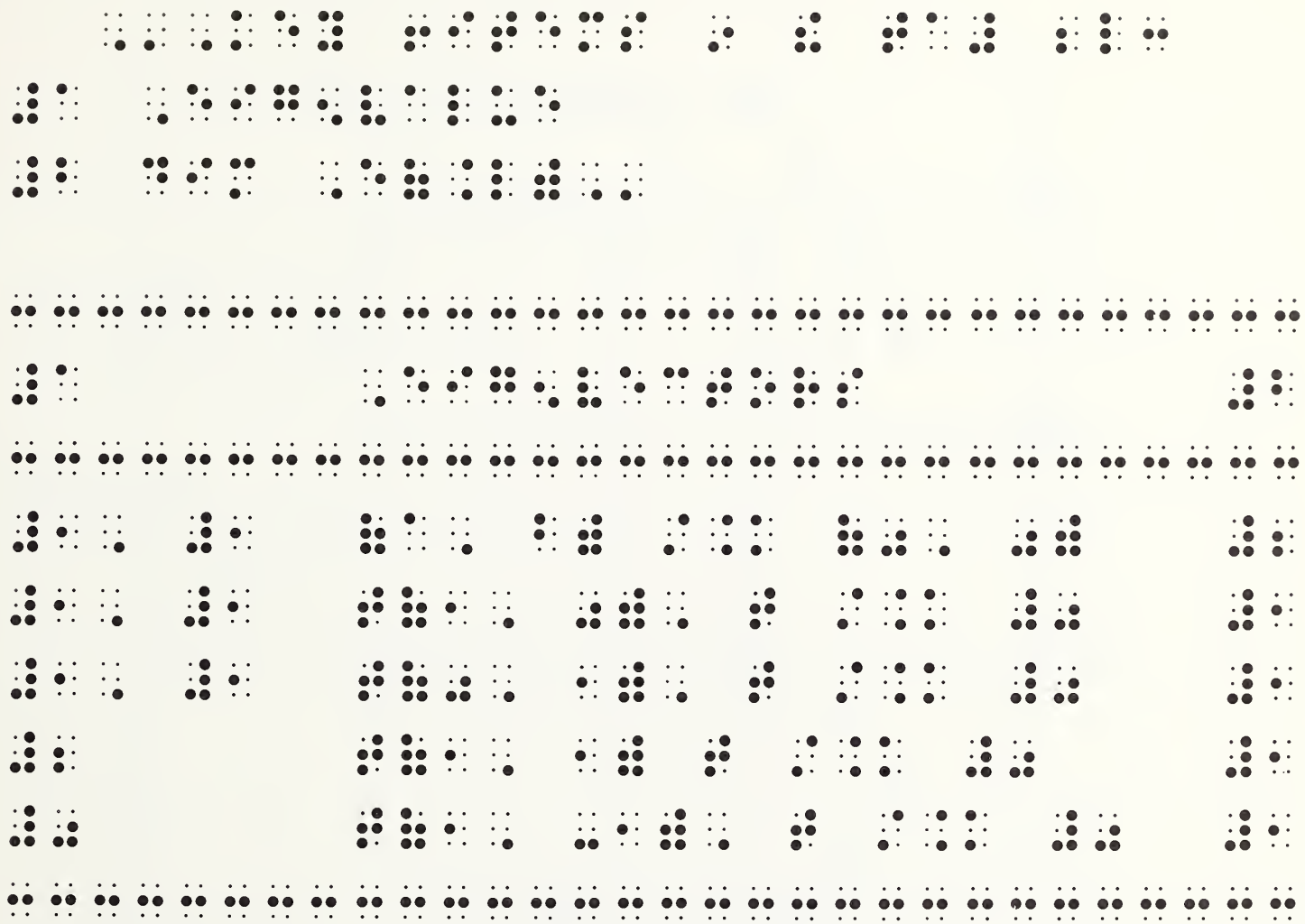
§186. **Keying:** A numeric or alphabetic key may be substituted where there is not enough space for determinants, matrices, column headings, table entries, figure labels, etc.

a. A numeric key should consist of consecutive numerals, beginning with number one, which should be written in the *upper* part of the cell and preceded by the numeric indicator but not punctuated. Identical items should be assigned the same key number.

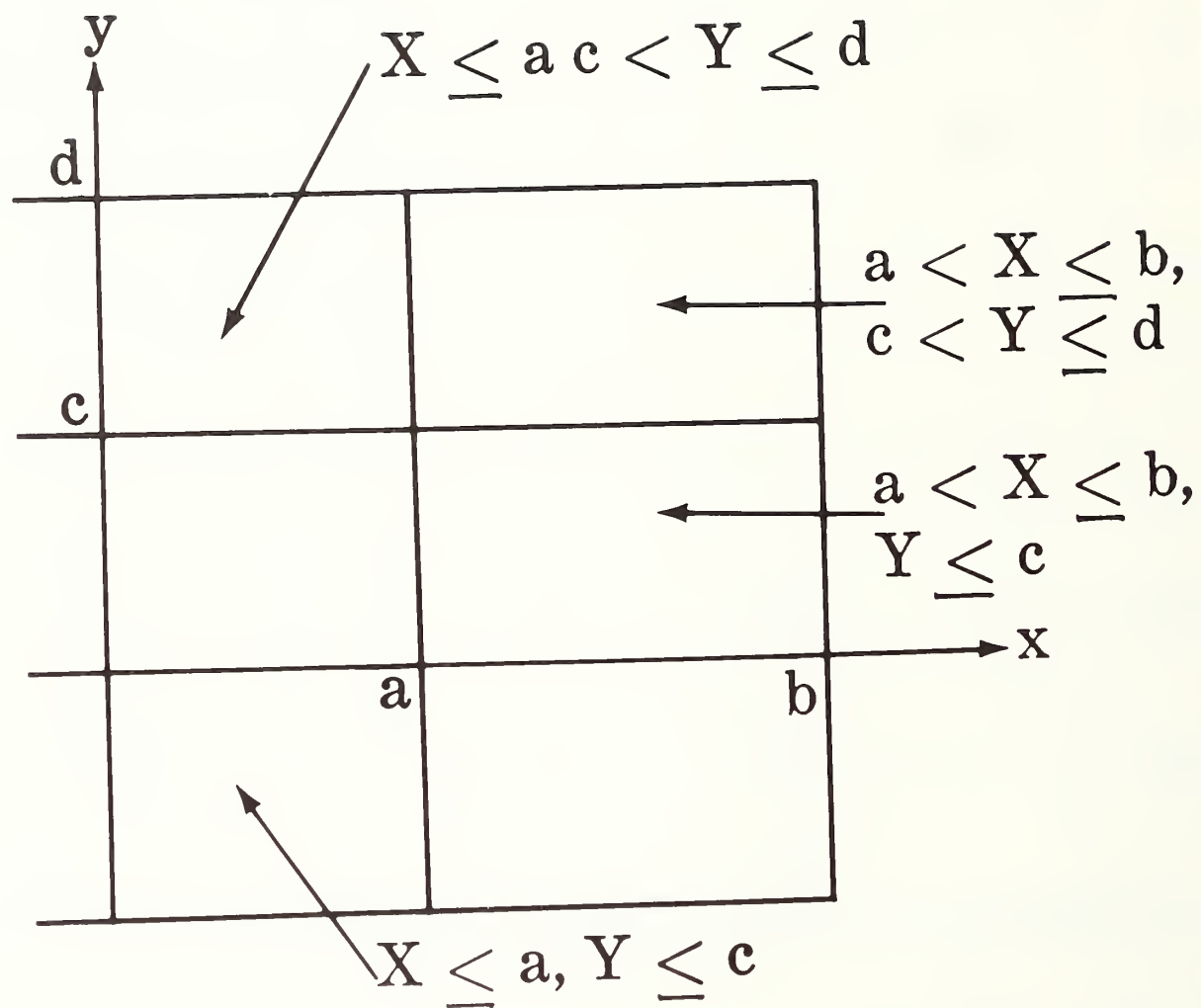
b. An alphabetic key should consist of two lower-case English letters suggestive of the item they represent, if possible. Identical items should be assigned the same key letters. The alphabetic key must not be used if the author uses two lower-case letters for entries in the table or for labels in the figure.

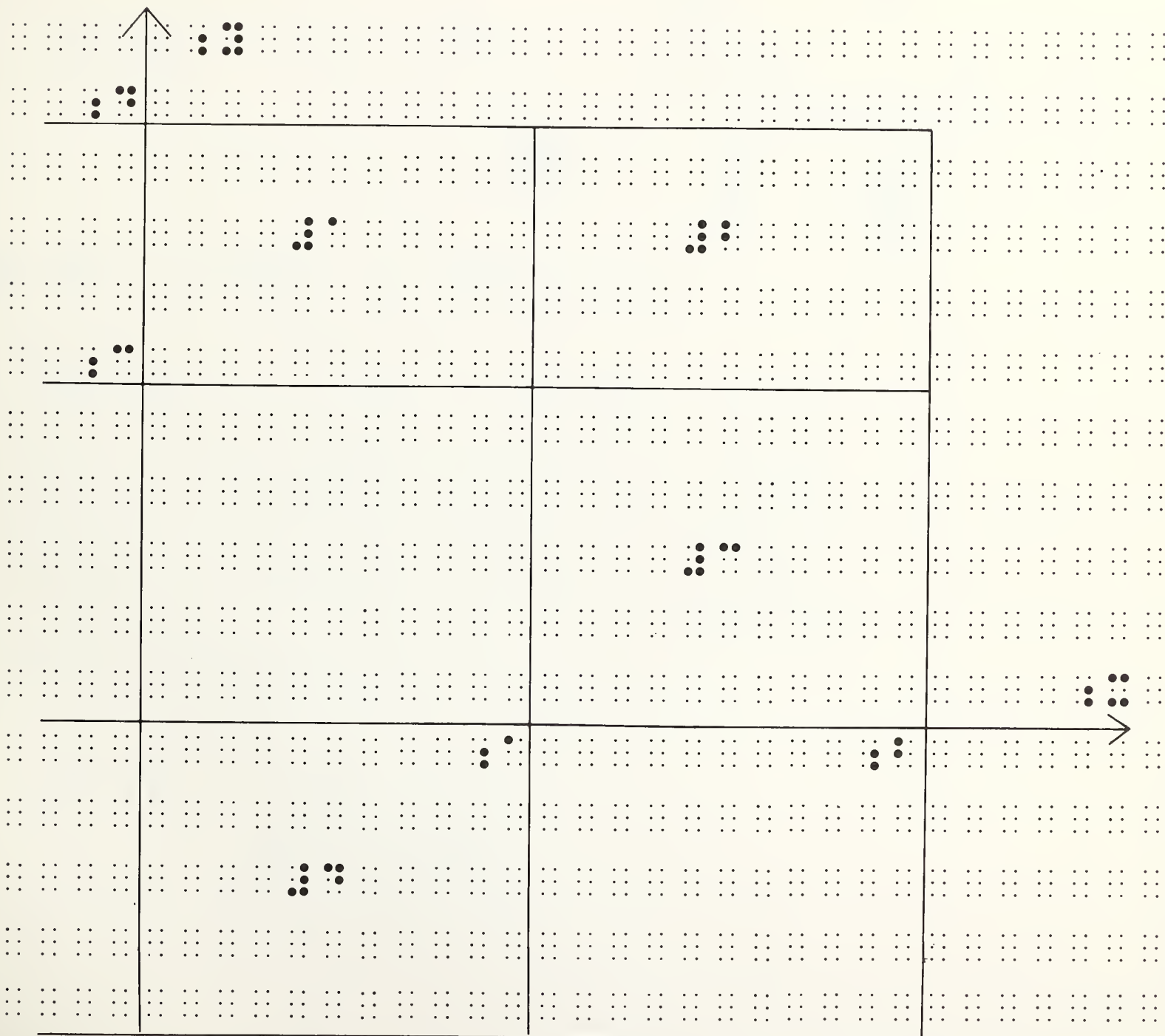
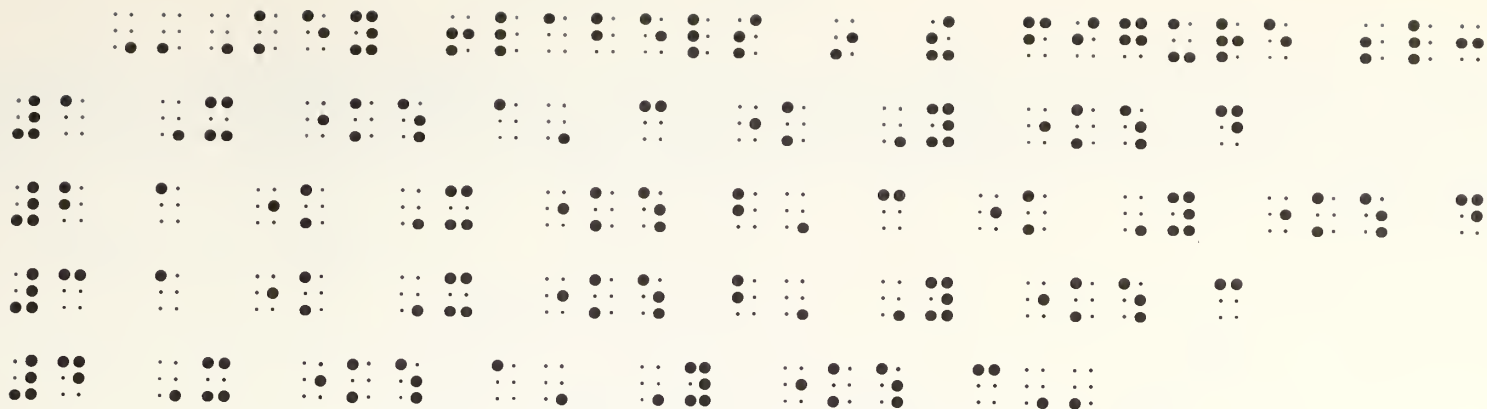
(2)

Eigenvalue	Eigenvectors	dim E (λ)
1, 1	(a, b) ≠ (0, 0)	2
1, 1	t(1, 0), t ≠ 0	1
1, 1	t(0, 1), t ≠ 0	1
2	t(1, 1) t ≠ 0	1
0	t(1, −1), t ≠ 0	1



(3)





FORMAT (CONTINUED)

§187. **Spatial Arrangements With Main Divisions and Subdivisions:** When spatial arrangements contain both main divisions and subdivisions, the first main division should begin at the margin, and the first subdivision should follow on the same braille line if there is no material between the main division number and the first subdivision number. As many subdivisions as possible should be placed on one line. If additional subdivisions are left, they should be transcribed beginning in cell 3, after leaving a blank line below the longest arrangement above.

(1)

Solve

2

$$\begin{array}{lll}
 1. \ a. & \begin{array}{r} 27.5 \\ \times 37.2 \\ \hline \end{array} & \begin{array}{l} b. \quad a^2 + b^2 \\ \times a + b \\ \hline \end{array} \quad c. \quad \begin{array}{r} \frac{3}{4} \\ \times \frac{1}{2} \\ \hline \end{array}
 \end{array}$$

2. Perform the operations.

$$\begin{array}{lll}
 a. & \begin{array}{r} 97 \\ 308 \\ 536 \\ 2748 \\ 59 \\ \hline \end{array} & \begin{array}{l} b. \quad .94 \\ .58 \\ .06 \\ .75 \\ \hline \end{array} \quad c. \quad \begin{array}{r} 37,120 \\ -28,934 \\ \hline \end{array}
 \end{array}$$

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HOMEWORK

Prepare the following exercise for submission to your teacher. Proofread carefully.

EXERCISE 16

1. Check.

$$\begin{array}{r} \overset{2}{\cancel{8}} \overset{11}{\cancel{1}} \\ - \quad 5 \\ \hline 2 \quad 6 \end{array}$$

$$\begin{array}{r} \overset{1}{\cancel{2}} \overset{12}{\cancel{2}} \\ - \quad 8 \\ \hline 1 \quad 4 \end{array}$$

$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{14}{\cancel{4}} \\ - \quad 6 \\ \hline 3 \quad 8 \end{array}$$

$$3 \frac{1}{5}$$

$$\begin{array}{r} \cancel{3} \overset{5}{\cancel{5}} \\ \hline 25 \end{array}$$

$$\begin{array}{r} 25 \overline{)80} \\ \underline{75} \\ 5 \end{array}$$

$$3. \quad (2 + \frac{a}{b}) \div (4 - \frac{a^2}{b^2}) = \frac{2b+a}{b} \div \frac{4b^2-a^2}{b^2} = \frac{\overset{1}{\cancel{(2b+a)}}}{\underset{1}{\cancel{b}}} \times \frac{\overset{b}{\cancel{b^2}}}{\underset{1}{\cancel{(2b+a)}}(2b-a)} = \frac{b}{2b-a}$$

$$\begin{aligned} 4. \quad & \frac{5x+15}{x^2-9} \div \frac{10x^2+10x}{4x-12} \\ &= \frac{5x+15}{x^2-9} \times \frac{4x-12}{10x^2+10x} \\ &= \frac{\overset{1}{\cancel{5}}(\overset{1}{\cancel{x+3}})}{\underset{1}{\cancel{(x+3)}}(\underset{1}{\cancel{(x-3)}})} \cdot \frac{\overset{2}{\cancel{4}}(\overset{1}{\cancel{x-3}})}{\underset{1}{\cancel{10x}}(x+1)} \\ &= \frac{2}{x(x+1)} \end{aligned}$$

5. Suppose your mother made a little cake for you and your friends. Into how many equal parts would you divide it if there were four of you to share the cake? What would each part be called?

$$\frac{1}{4} \quad \begin{array}{l} \text{the number of parts each child receives} \\ \text{the total number of parts} \end{array}$$

$$\begin{array}{r} r^2-4s^2 \\ \hline s^2 \\ \hline r+2s \\ \hline s \end{array}$$

$$\begin{array}{r} 6. \quad \begin{array}{r} 4r-2s^2 \\ \hline 3s \\ \hline 2s^2-3r \\ \hline 4r \end{array} \end{array}$$

7. Check for correctness.

$$\sqrt{3} = 1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}$$

8. Solve and check.

$$\begin{cases} 3c = 4d + 17 \\ 2c + 3d = 0 \end{cases}$$

9. The system

$$\begin{cases} x + 2y = 8 \\ 2x - 3y = 2 \end{cases}$$

has the solution set $\{(4, 2)\}$.

10.
$$\begin{cases} x + y - 5 = 0 \\ 4x - y - 10 = 0 \end{cases}$$

11. Solve.

$$\begin{cases} x + y + 3 = 0 \\ x - y + 5 = 0 \end{cases}$$

12. Solve and check.

$$\begin{cases} x - 2y + 12 = 0 \\ 4x + y + 3 = 0 \end{cases}$$

13. That $(0, 0)$ is the identity element for vector addition is shown below.

$$\begin{aligned} (v_1, v_2) + (0, 0) &= (v_1 + 0, v_2 + 0) = (v_1, v_2) \\ (0, 0) + (v_1 + v_2) &= (0 + v_1, 0 + v_2) = (v_1, v_2) \end{aligned} \quad \text{For all real numbers } v_1 \text{ and } v_2.$$

14. If $A = \begin{bmatrix} x & 3 \\ 2 & y \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$, find the values of x and y so that $A^T = B$.

15.
$$\begin{bmatrix} u_1 & v_1 \\ u_2 & v_2 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ -7 & 5 \end{bmatrix}$$

16.
$$\begin{vmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_r \end{vmatrix} = \begin{vmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ b_{31} & b_{32} & \dots & b_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{r1} & b_{r2} & \dots & b_{rn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{vmatrix}$$

17.
$$dT = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \dots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \dots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}$$

18. The Jacobian determinant of the mapping is

$$J(\rho, \theta, \phi) = \begin{vmatrix} \cos \theta \sin \phi & \sin \theta \sin \phi & \cos \phi \\ -\rho \sin \theta \sin \phi & \rho \cos \theta \sin \phi & 0 \\ \rho \cos \theta \cos \phi & \rho \sin \theta \cos \phi & -\rho \sin \phi \end{vmatrix} = -\rho^2 \sin \phi.$$

19. Can you make a vertical bar graph using the material given in this table?

City	Population	Telephones
New York	7,781,984	4,411,982
Chicago	3,550,404	1,894,012
Los Angeles	2,479,015	1,051,396
Philadelphia	2,002,512	1,083,041

20. Write the linear equations expressing the relations between x and y shown in the following tables:

a.

x	y
1	1
2	4
3	7
5	13

b.

x	y
-4	-1
-3	3
-1	11

c.

x	y
-8	-6
-12	-8
-14	-9

Use the following array to name all the equivalent fractions for: $\frac{1}{1}, \frac{1}{4}, \frac{2}{3}, \frac{3}{6}, \frac{1}{6}, \frac{0}{4}, \frac{5}{6}, \frac{1}{3}, \frac{3}{4}$.

0												1
0												1
1												1
0						1						2
2						2						2
0				1		3			2			3
3				3					3			3
0			1			2			3			4
4			4			4			4			4
0		1		2		3		4		5		6
6		6		6		6		6		6		6
0	1	2	3	4	5	6	7	8	9	10	11	12
12	12	12	12	12	12	12	12	12	12	12	12	12

Calculate the eigenvalues of each of the matrices.

21. (a) $\begin{vmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -7 & 1 \end{vmatrix}$

(b) $\begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 3 & 20 \end{vmatrix}$

(c) $\begin{vmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{vmatrix}$

22. (a) $\begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}$

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